Edgenuity™

Principles & Practices for Mathematics

How Edgenuity Mathematics Courses Align with Research on Effective Instruction

June 2013

A Summary of Independent Research

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Introduction

Decades of research corroborate the importance of mathematics to academic and career success (National Commission on Excellence in Education, 1983; National Research Council, 2012). Early math learning is one of the strongest predictors of high school achievement (Siegler et al., 2012), while students who complete Algebra II in high school more than double their chances of earning a two- or four-year degree (Adelman, 2006). In addition, students who do better in mathematics earn more and have greater earnings growth (Murnane et al., 1995; National Mathematics Advisory Panel, 2008; Rose & Betts, 2004).

While there is widespread agreement on the need for solid mathematics skills, U.S. students' mathematical competence continues to be a great concern. According to the 2009 and 2011 National Assessment of Educational Progress (NAEP), only 38 percent of 12th grade students and 35 percent of 8th grade students scored at or above the proficient level in mathematics. Consequently, educators are redoubling their efforts to improve the way mathematics is taught. Many are working to ensure students achieve higher levels of math learning by focusing on fewer topics, in more depth. Rather than including the same topics year after year, they are connecting and introducing new topics from grade to grade. They are also making instruction more rigorous by incorporating in the mathematics curricula more higher-order thinking skills, such as making predictions, reasoning abstractly, constructing arguments, solving complex problems, and discerning patterns.

Increasingly, schools and districts are turning to online and blended instructional models to meet the needs of a broad population of students. Research supports the conclusion that technology-enhanced learning can simultaneously improve self-regulation and mathematical skills (Bernacki et al., 2011; Cheung & Slavin, 2013; Kramarski & Gutman, 2006; Kramarski & Mizrachi, 2006; Wenglinsky, 1998). Moreover, data show that students can learn just as well from online instruction as from traditional classes (Cavanaugh, 2013). However, simply putting students in front of a computer is not a guarantee of improved mathematics achievement. To maximize learning, online mathematics instruction needs to be research-based and practice-tested.

Edgenuity is committed to developing innovative mathematics courses that are grounded in research and best instructional practices. Edgenuity mathematics courses are based on four well-established, evidence-based principles:

- Courses directly address students' misconceptions.
- Courses provide systematic and explicit instruction, designed to help students acquire, practice, and apply skills and knowledge.
- Courses promote deep content learning in numbers and operations, algebra, statistics and probability, and geometry.
- Courses develop students' ability to make sense of problems, and teach them to reason abstractly and quantitatively, construct viable arguments, model with mathematics, use appropriate tools, attend to precision, look for and make use of repeated reasoning and structure.

This report provides an overview of the Edgenuity approach to instruction—and then goes on to describe in detail how Edgenuity translates the best mathematics research in online learning, neuroscience, pedagogy, educational psychology, and instructional design into its mathematics courses.

Edgenuity Overview

Since 1998, Edgenuity has been providing online courses to propel student achievement. Edgenuity now offers more than 100 courses for grades 6–12 in English language arts, social studies, math, science, world languages, and general electives. Edgenuity also offers a full suite of career pathways and electives, test preparation solutions for national and state exams, and credit recovery courses. Edgenuity courses can easily be implemented in alternative education settings that require flexible scheduling.

Edgenuity courses are designed to meet a range of district needs using flexible implementation models. For example, some schools and districts implement a blended learning model for Edgenuity courses. In this model, students spend part of their time completing Edgenuity online coursework. The rest of their time is spent in workshops, projects, or small-group instruction led by local teachers. These teachers provide tutoring, review quizzes, assign grades, and confirm that students are actively engaged in courses, to ensure that students achieve concept mastery and can apply what they've learned.

In other models, students complete the Edgenuity coursework online, and also receive guidance from a highly qualified live online teacher from Edgenuity. The live online teacher assesses student work, provides feedback, and offers additional support, instruction, and tutoring as needed via chat, e-mail, and phone calls.

Course Design Process

Edgenuity courses are created by cross-functional teams of experienced educators and instructional course designers with expertise in curriculum development, instructional technology, and content-area education. To create a new course, the team begins with a careful analysis of state and national standards, as well as syllabi and curriculum maps of existing courses from exemplary, high-performing districts. The scope and sequence of a course is then created and reviewed by domain experts and education practitioners.

Using the principles of backward design, the team outlines each unit of instruction to capture big ideas and essential questions, refine learning objectives and lesson questions, and document anchor assessments and tasks. Prototype lessons are drafted and team-reviewed against research-based best practices, the iNACOL National Standards for Quality Online Courses, Common Core State Standards, and Edgenuity's own development rubrics and guidelines before the remainder of the lessons are created.

Instructional Model

Edgenuity courses reflect research-based instructional practices to meet the needs of all students. Courses feature rigorous, explicit instruction led by certified on-screen teachers. Motivating, media-rich content keeps students engaged, and powerful interactive instructional tools help them build content knowledge and essential skills. Aligned to Common Core and other state standards, Edgenuity's courses include challenging content, relevant activities, adaptable formative and summative assessments, and real-time feedback.

Expert on-screen teachers present learning objectives, explain concepts, model strategies, and provide relevant examples that help students transfer knowledge and make real-world connections. Meaningful assignments ensure students master key concepts and develop analytical and critical thinking skills. Students complete a range of tasks—including independent reading, practice, and guided online exploration, as well as projects and performance tasks. Simulations and virtual labs help students make and test predictions, while graphics, images, and animations bring content to life.

Each lesson includes assessments to determine whether students have mastered the lesson objectives. Cumulative practice and assessment is included at the end of each unit or topic, as well as at the end of each semester.

Interactive Tools and Supports

A full suite of digital tools helps students access content, complete assignments, and build essential skills. For example, animations and simulations provide explanation and modeling of key concepts and processes. Digital highlighters and sticky notes help students organize information, ask questions, and record observations. Read-aloud and translation tools help English language learners and students with special needs

A digital notebook called eNotes includes an equation editor so students can take notes, record, synthesize, and organize their thinking in mathematics courses. A dynamic glossary and word look-up tool helps students build their academic vocabulary, while transcripts and video captions enable students to follow along with the on-screen teacher. Calculators, graphic organizers, and other tools help students complete assignments and promote the deep transfer of knowledge and skills.

Learning Management System Features

Edgenuity's learning management system offers a number of tools and features to support effective implementation. These include:

- Customizable assessment settings for time allotted to complete tests, grade weights, number of retakes, and passing thresholds
- Clear graphical representations of student progress to help students stay on pace
- A customizable assignment calendar to help students track the coursework they should be completing each day
- Diagnostic and prescriptive capabilities to individualize student learning paths based on existing levels of mastery
- Robust reporting to enable educators to monitor student engagement, progress, and achievement
- Administrator tools to set teacher permissions, review teacher actions, and monitor student data
- A web-based Family Portal to enable parents and guardians to monitor student learning from their computer, tablet, or smartphone

Addressing Misconceptions

Research suggests that students' understanding of mathematics is shaped largely by their previous experiences and everyday knowledge (Fuson et al., 2005). Students' prior knowledge, however, can be incomplete and may lead to errors (Groth, 2013; Smith et al., 1993; Swan, 2005). For example, students sometimes believe the numerators and denominators of fractions can be treated as separate whole numbers (Ashlock, 2010). Students who make this error misapply their knowledge of whole numbers and add or subtract the numerators and denominators of two fractions (e.g., $\frac{1}{8} + \frac{9}{8} = \frac{10}{16}$ or $\frac{4}{7} - \frac{1}{3} = \frac{3}{4}$). This misconception is particularly counterproductive to learning, because "understanding fractions is essential for algebra and other more advanced areas of mathematics" (Siegler et al., 2010).

Experts agree that effective instruction should review prerequisite learning (Rosenshine, 1995), expose misconceptions (Swan, 2005), and guide students toward effective strategies and more advanced understanding (Fuson et al., 2005; Swan, 2001). To help students address faulty assumptions, students must make their thought processes transparent and compare their responses to other responses (Swan, 2001).

Edgenuity mathematics courses are designed to build from learners' background knowledge and directly address their misconceptions to strengthen conceptual understanding. This section addresses how Edgenuity activates students' prior knowledge and addresses students' misconceptions to foster mathematical learning.

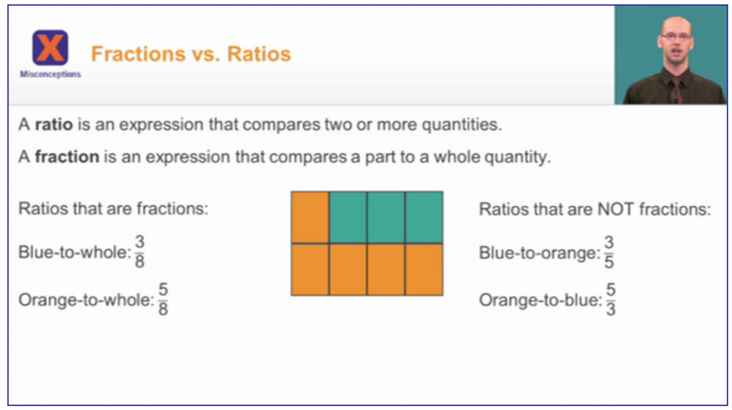
How Edgenuity Addresses Students' Misconceptions

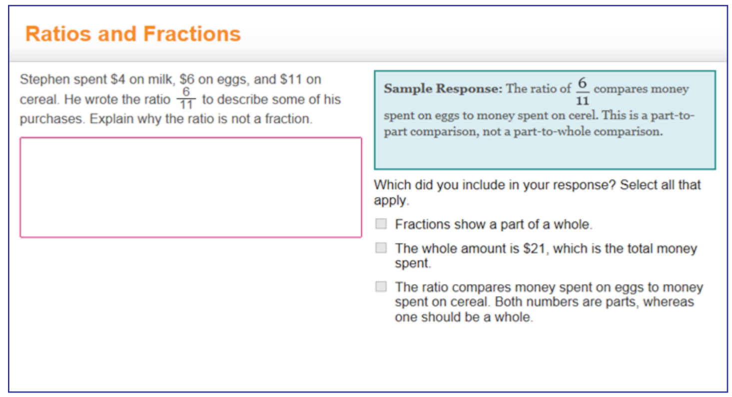
Edgenuity mathematics lessons are constructed to activate prior knowledge and clarify common misconceptions.

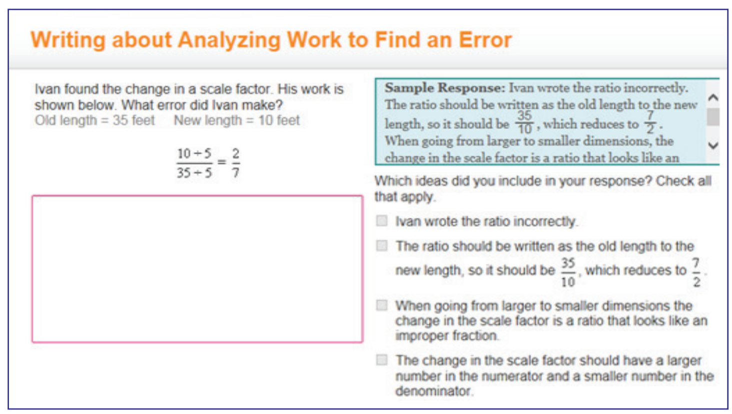
Before direct instruction begins, the onscreen teacher reviews skills and knowledge students will need to understand the lesson and complete activities. Edgenuity lessons also connect what students are learning with relevant prior knowledge by using a warm-up activity that introduces students to the lesson's topic. For example, an Algebra II lesson on functions and relations uses the relationship between states and capitals to introduce the concept of ordered pairs. Students' prior understanding that each state has only one capital helps them understand one-to-one functions in which each element in the domain is related to one specific element in the range.

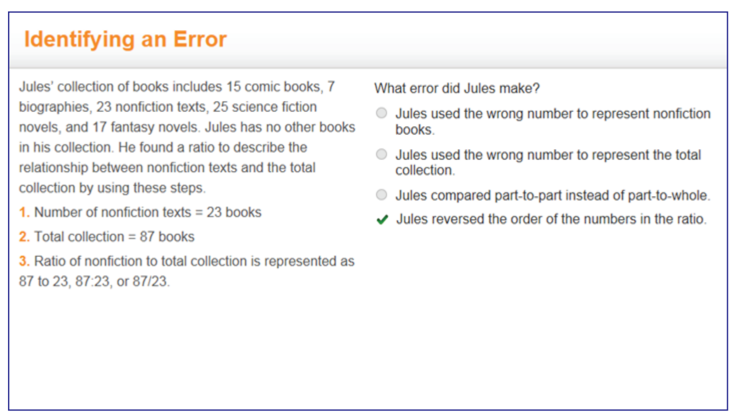
Misconceptions are also explicitly addressed during direct instruction. Throughout lessons, on-screen instructors provide clear, detailed explanations about common misconceptions (with examples and non-examples) and explain how to avoid errors or interpret concepts correctly—thereby strengthening students' conceptual understanding. For particularly entrenched misconceptions, Edgenuity provides students with a visual cue designed to focus their attention on a common error. This "misconception" icon reminds students to check their own interpretation of a difficult topic. For example, students often wrongly conflate ratios with fractions because they use the same notation. In a grade 6 lesson on ratio notation, students learn that while a ratio compares two or more quantities, a fraction compares a part to a whole. The on-screen teacher uses verbal explanations and visual representations to further clarify this concept.

Edgenuity lessons require students to explain and justify their reasoning, identify errors and inconsistencies in sample work, use built-in checklists to evaluate reasoning, and compare sample responses. For example, in a grade 6 lesson on ratio notation, students must explain why a particular ratio is not an example of a fraction. They are then asked to identify an error in a sample problem and write about a hypothetical student's faulty reasoning in a change of scale problem.









At the end of the lesson, the on-screen teacher summarizes what has been learned, drawing out common misconceptions and explicitly discussing them.

Explicit Instruction

Explicit instruction, often called systematic instruction, is a highly structured, clear-cut approach to teaching mathematics (Archer & Hughes, 2011; Stein et al., 2006). Decades of experimental and correlational studies show that explicit instruction can help improve students' mathematics achievement, graduation rates, and college acceptance rates (Baker et al., 2002; Gersten et al., 2008; Kozieff et al., 2000; Rosenshine, 1995; Vaughn et al., 2012). Educational researchers agree there are 10 components of explicit instruction that promote mathematical learning. These are:

- Clearly defined learning objectives and goals
- Unambiguous explanations of conceptual and procedural knowledge
- Small, manageable chunks of instruction
- Multiple representations
- Precise vocabulary instruction
- A wide array of models, demonstrations, examples, and non-examples
- Real-world applications
- Scaffolded practice with prompts
- Regular feedback and checks for understanding
- Cumulative review over time

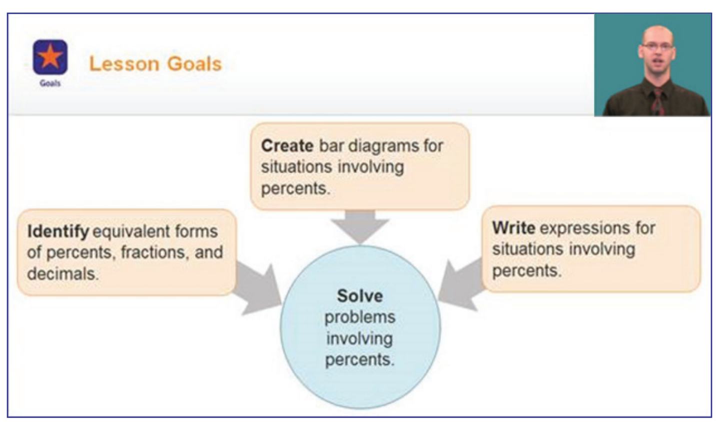
Explicit instruction is a cornerstone of Edgenuity mathematics lessons. This section addresses how Edgenuity translates the research behind explicit instruction into instructional practice.

Clearly Defined Learning Objectives and Goals

Research shows there is a strong correlation between clarity of lesson objectives and mathematics achievement (Hollingsworth & Ybarra, 2009; Swan, 2003). Consequently, experts and educational researchers agree that setting clear goals and expectations for learners is essential to improve students' mathematical learning (National Research Council, 2012; Stein et al., 2006).

How Edgenuity Mathematics Courses Support Clearly Defined Learning Objectives and Goals

All Edgenuity lessons begin by presenting clearly defined instructional objectives and articulating what students will be expected to learn and do as part of the lesson. Lesson goals are written in student-friendly language and are directly connected to assignments and tasks. They are often presented in a graphic organizer or table to make relationships between concepts and skills apparent to students.



Balance Conceptual and Procedural Knowledge

Empirical research suggests that mathematical learning is shaped by conceptual and procedural understanding. Conceptual knowledge is the connected web of core concepts, representations, and relations that define a particular domain (Rittle-Johnson, Schneider, & Alibali, 2001). Students who have solid conceptual understanding are able to pinpoint the characteristics of concepts as well as manipulate, interpret, and apply them to novel problems in new settings. Procedural knowledge consists of the rules, symbols, and procedures needed to solve problems or complete tasks (Hiebert & Lefevre, 1986).

There is widespread agreement that conceptual and procedural knowledge are positively correlated and that students learn the two simultaneously (Hiebert & Lefevre, 1986; Kloosterman & Gainey, 1993; Rittle-Johnson & Siegler, 1998). As such, students "who can connect mathematical procedures with underlying concepts are likely to apply those procedures correctly rather than in an arbitrary, inappropriate fashion" (Kloosterman & Gainey, 1993, p.10). Effective instruction should first address the meaning of important mathematical ideas and the connections linking these ideas, before teaching procedures (Rakes et al., 2010).

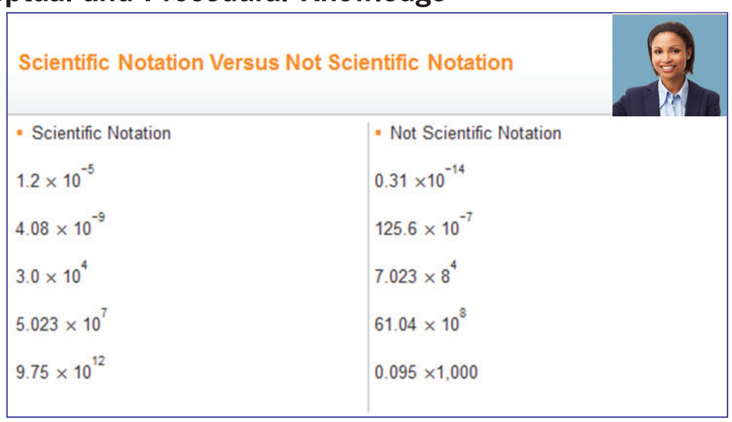
Research-based strategies for building conceptual and procedural knowledge include the following:

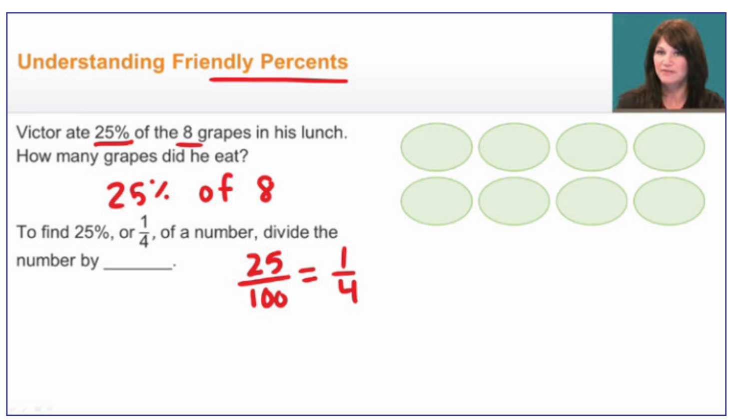
- Clear, detailed, and explicit teaching of concepts, information, and rules (Archer & Hughes, 2011; Hiebert & Lefevre, 1986)
- Calling attention to the characteristics of and similarities, differences, and connections among concepts (Yetkin, 2003)
- Clear, step-by-step directions (Rosenshine & Stevens, 1986)
- Extensive use of appropriate examples, non-examples, and worked examples (Archer & Hughes, 2011; Stein et al., 2006)
- Helping students learn not only how but also when—under what circumstances and conditions—to apply specific factual knowledge, strategies, and procedures (Fuson et al., 2005; NRC, 2000; NRC, 2012).

How Edgenuity Mathematics Courses Balance Conceptual and Procedural Knowledge

Edgenuity believes robust conceptual and procedural understanding leads to deep mathematical learning. It is only when students clearly understand concepts and can apply them in a variety of settings that mathematics becomes comprehensible. The following are part of every Edgenuity mathematics course:

- Clear, detailed explanations are provided for the concepts, information, and rules students are expected to learn.
- In presenting information, on-screen teachers provide an overview of the concept, highlight key characteristics, and draw students' attention to the structure of the information.
- Concepts and rules are explained with examples and non-examples. In an algebra lesson about linear equations, for example, students learn that the standard form is one way to express an equation of a line ($Ax + By = C$). Together with the definition, examples and non-examples of the standard form are provided. For example, in an algebra lesson, the on-screen instructor shows that the equation $3x + 5y = 3$ is in standard form, but the equation $2y = 4x + 2$ is not in standard form. In a grade 8 lesson, the on-screen instructor explains what constitutes scientific notations, with a variety of examples.





- On-screen teachers deliberately show how different concepts connect to one another. For instance, a grade 6 lesson asks students to determine how many grapes a boy consumed if he ate 25 percent of the 8 grapes in his lunch. Rather than telling students to multiply 0.25 × 8, the on-screen teacher promotes conceptual understanding by discussing how 25% is the same as ¼, which means the number 8 can be divided by 4. Therefore, the student ate 2 grapes.

- Assignments require students to compare, contrast, generate, and integrate related concepts and principles, as well as interpret and apply the signs, symbols, and terms used to represent concepts. For example, in grade 6 lessons students are asked to identify the criteria for using multiplication or division when converting units in the customary measurement system. Students must show why multiplication is used to convert larger units to smaller units, as well as why division is used to convert smaller units to larger ones. Another lesson requires students to compare ratios and fractions to expressions and equations. In grade 7 lessons, students compare experimental and theoretical probability as well as expressions and equations.
- Instruction and assignments frequently remind students of "big-picture" connections to major themes and ideas they have been learning about in the mathematics course. For example, in an Algebra I lesson, students learn that the goal of solving linear equations is to figure out an unknown. To isolate a variable, students are taught to use the multiplication property of equality, the addition property of equality, the subtraction property of equality, the division property of equality, and order of operations. In doing so, students learn the underlying principle of balance and equality—that whatever operations are done on one side of the equation must also be done on the other side.
- Students are taught when to use specific procedures and how and when to apply facts and information. For example:
    - In an Algebra II lesson, students learn to use graphing and elimination methods for solving systems of equations. The on-screen instructor coaches students to use the elimination method when there are decimals that are hard to graph.
    - In a grade 6 math unit on dividing fractions, the on-screen instructor teaches a number of meaning-based strategies instead of the traditional "flip and multiply" algorithm. When the algorithm is introduced, it is built conceptually using repeated reasoning.
- Interactive simulations, graphs, measurement tools, number lines, algebra tiles, and probability simulators help students develop procedural fluency.

Small, Manageable Segments of Instruction

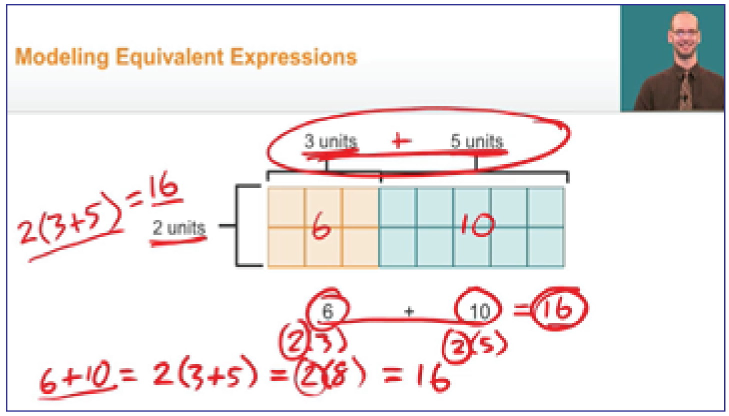
Neuroscientists have identified two main types of memory: short-term memory, also known as working memory, where we consciously process information; and permanent long-term memory with a much larger body of connected information. After information has been organized and stored in long-term memory, it can be accessed again as needed without placing a large burden on working memory (Sweller, 2008, p.373). The knowledge in long-term memory is used to understand new memories.

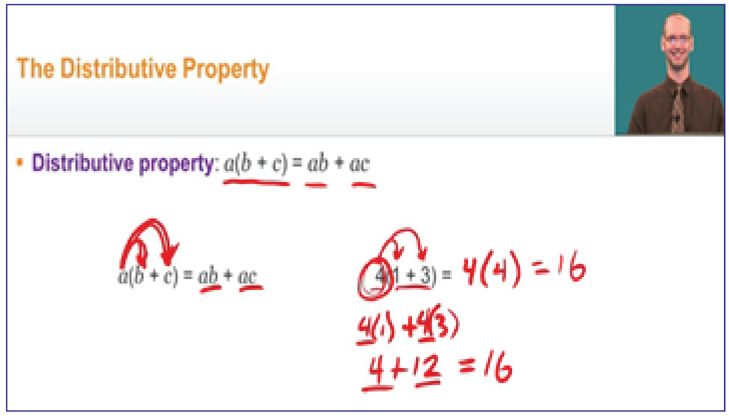
Research indicates that human short-term memory has capacity and time limitations. Scholars estimate the brain can accommodate only seven pieces of information (plus or minus two) at a time (Sousa, 2008). In addition, adolescents can only "process an item in working memory intently for 10 to 20 minutes before fatigue or boredom with that item occurs and the individual's focus drifts" (Sousa, 2008, p.52). When more than seven pieces of information are presented at a time, students cannot process the information. Because of this, experts recommend that instruction be taught in small segments (Sweller, 2008).

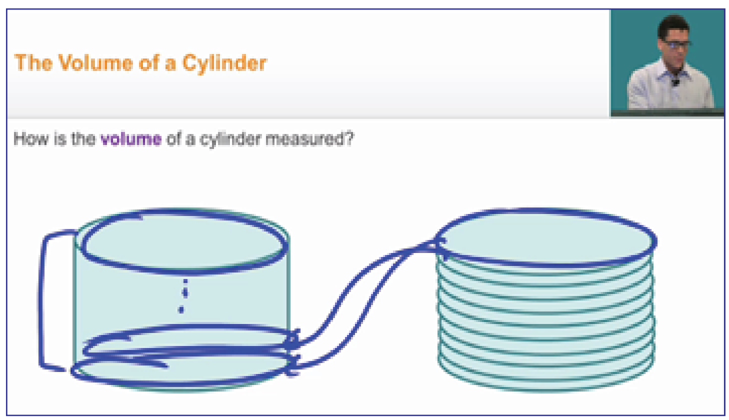
How Edgenuity Breaks Instruction into Small, Manageable Segments

On-screen instructors teach students new pieces of information in small, manageable segments. Overall concepts and skills are broken down into smaller components. Students practice each key step separately, then practice all the steps together to synthesize learning. For example:

- Throughout Edgenuity mathematics courses, the distributive property is introduced and reviewed. In grade 6 lessons, on-screen teachers explain that the distributive property means the sum of two numbers is the same as its parts. This means you get the same answer if you multiply a number by a group of numbers added together, or if you multiply each separately and then add them. Students first use an area model where they see that 2 squares of (3 + 5) is the same as 2 squares of 3 plus 2 squares of 5 is equivalent to the total area (16). On-screen instructors explain why 2 can be "distributed" across the 3 + 5, into 2 × 3 and 2 × 5. They then practice applying the distributive property to expressions and equations in a wide array of contexts. Eventually, students synthesize the information while answering specific word problems.
- In a grade 8 geometry unit, students learn that certain objects retain their shape, size, and angle measures under transformations. Students first learn what rotation, reflection, and translation are. They then analyze whether an object changed shape under transformation.
- Similarly, in a lesson on volume, on-screen instructors develop formulas for the volumes of solids based on the formulas for the areas of their bases. Students then use what they know about simple solids to find the volumes of composite solids.
- In an Algebra II lesson about solving quadratic equations by factoring, students first learn to write the equation in standard form, so that one side is set equal to zero. They then learn how to factor the expression. Next, they practice using the zero product property to write and solve linear equations. Finally, they practice checking their work and solutions. Eventually, students practice the complete process without being prompted step-by-step.







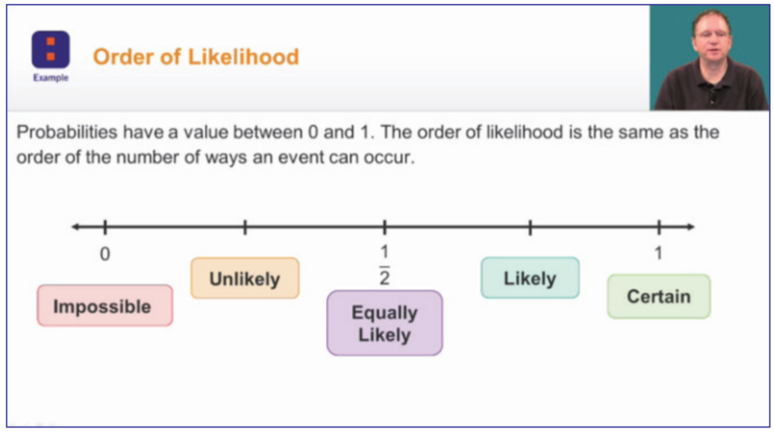
Multiple Representations

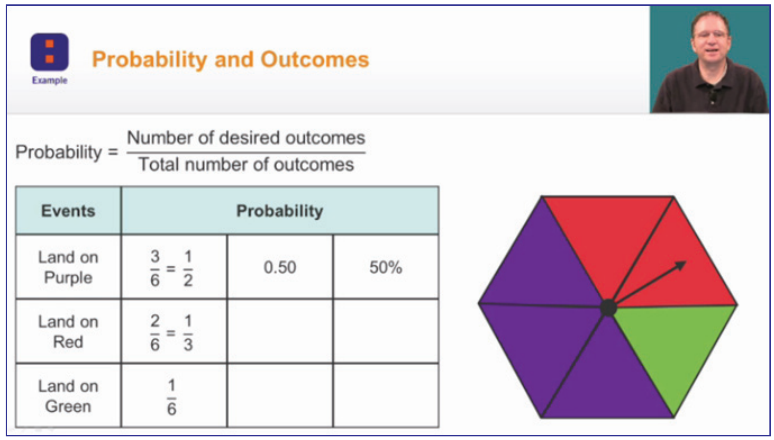
Because "[l]earners differ in the ways that they perceive and comprehend information that is presented to them," experts recommend that instruction and supports for students be presented in multiple ways (CAST, 2011, p.14). In fact, research confirms that students' numeric, algebraic, statistical and probabilistic, geometric, and data skills can be enhanced when concepts are presented in a variety of contexts (Groth, 2013). Kloosterman (1993, p.11) finds that "seeing fractions represented with pattern blocks, fraction bars, fraction circles, and rods help students to understand the concept of fraction, independent of its physical representation." Similarly, Watson (2007) notes that electronic algebra tiles, blocks, spatial representations, and bar diagrams have been useful for helping students develop schema to organize their learning. Groth agrees, adding, "[S]tudents who hold multiple meanings of functions and expressions were better able to see them as processes which could be combined as compound functions" (p.212).

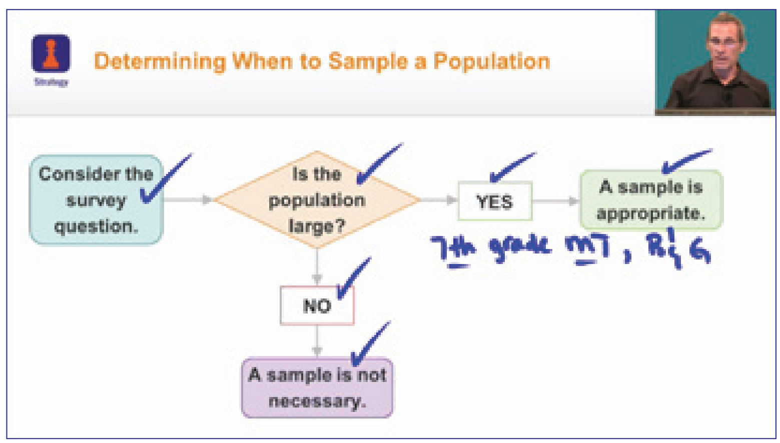
Graphic organizers that present information in a visual format have been shown to help students organize new knowledge and improve mathematical learning (Nesbit & Adesope, 2006, p.427). Researchers posit that graphic organizers may reduce students' cognitive load by drawing attention to key elements that connect known knowledge to new knowledge (Mayer, 2011; Nesbit & Adesope, 2006, citing Larkin & Simon, 1987).

How Edgenuity Mathematics Courses Use Multiple Representations

To accommodate students of all learning styles and diverse abilities, Edgenuity mathematics courses present key information and concepts using multiple formats, including video direct instruction, verbal modeling, pictures, symbols, dynamic representations, graphs, and text (with optional read-aloud support). On-screen teachers point out how aspects of each representation relate to the others. Then, students articulate which representation is most efficient for describing different phenomena. For example:







- In a grade 6 lesson, students use fraction bars, pattern blocks, fraction circles, moveable pie pieces, number lines, shaded area models, and arrays to visually demonstrate how fractions, decimals, and percents can be equivalents. At the end of the lesson, students compare and explain the usefulness of different representations.
- In a grade 7 lesson, students use symbols, words, and an interactive tool with a table and graph to model proportional relationships. Students plot points on a graph and see that a line is created only when the ordered pairs are all proportional. Students then answer questions in which they compare the representations. In another grade 7 lesson, students use line graphs, tables, visual diagrams, and simulations to predict the probability of different events.
- In a grade 8 lesson, students learn the characteristics of linear and nonlinear functions by exploring interactive graphs, graphing calculators, scatter plots, equations, tables, and concrete models.
- In an Algebra II lesson, a simple simulation helps students understand the difference between linear and exponential growth. To help make this abstract idea more concrete, students play two games with different scoring systems to see how their scores change over time with linear and exponential growth. They then model the games with tables and equations.

A wide variety of graphic organizers is included in instruction, tasks, and assignments.

Vocabulary Instruction

There is broad consensus that vocabulary plays a critical role in students' mathematical development (Groth, 2013; McREL, 2010; Miller, 1993; Monroe & Orme, 2002; Rubenstein, 2007; Smith & Angotti, 2012). As Miller (1993) points out, "[W]ithout an understanding of the vocabulary that is used routinely in mathematics instruction, textbooks, and word problems, students are handicapped in their efforts to learn mathematics" (Monroe & Orme, 2002, citing Miller, p.312).

Studies indicate that three significant challenges impede mathematical vocabulary development (Monroe & Orme, 2002; Rubinstein, 2007; Smith & Angotti, 2012). First, many words are shared between mathematics and English but have different meanings (e.g., median, product, prime). Second, words can represent entirely new concepts with which students have no experience. Third, the meaning of words can vary among the various disciplines of mathematics (e.g., ray, range, square). As Smith & Angotti (2012, p.44) point out, "[M]athematics vocabulary is unique in that many words have both generic and specific meanings, while at the same time key terms must be defined in a precise manner."

Educators and researchers agree that both broad and specialized vocabulary associated with mathematical topics (such as perfect numbers, quadratic equations, cosine, and mode) must be taught directly (McREL, 2010; Archer & Hughes, 2012).

As Archer & Hughes' (2012) review of the literature reveals, effective vocabulary instruction should not focus on students looking up the word in a dictionary and writing down definitions, but should provide clear presentations of word meanings with contextual examples, present multiple exposures to target words, provide contextual and definitional information, and actively engage in activities that foster word learning.

Graphic organizers that provide a definition, list characteristics, and provide examples and non-examples of concepts can also aid learning (Groth, 2013; McREL, 2010).

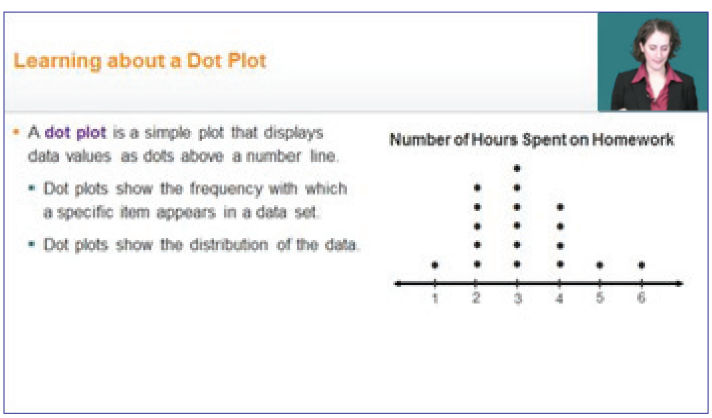
How Edgenuity Mathematics Courses Provide Direct Vocabulary Instruction

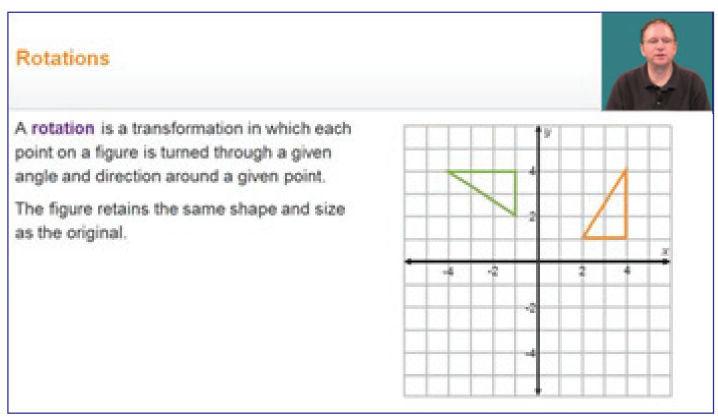
Edgenuity explicitly teaches and spirals academic and domain-specific vocabulary throughout its mathematics courses. Academic vocabulary words are carefully selected from the Averil Coxhead Academic Wordlist (2000) and Magali Paquot Academic Keyword List (2010). Specialized and technical domain-specific words are chosen based on the lesson content in which they appear.

In Edgenuity lesson warm-ups, students preview four to six academic (e.g., number, angle, and equation) and domain-specific (e.g., perfect square, quadratic equations, cosine, and mode) vocabulary words in the Words to Know slide.

During instruction, on-screen teachers define and teach critical words. Explanations of vocabulary provided in Edgenuity are clear, easy to understand, and illustrate not only what the words mean but also how they are used. On-screen instructors describe words' meanings, critical attributes of words, as well as providing contextual and definitional information. They also model the use of vocabulary for students—ensuring multiple exposures to high-yield words.

After instruction, students complete tasks to help them solidify their knowledge. They engage in activities that require them to identify non-examples and examples, explain phenomena using descriptive patterns, complete concept definition maps, and submit word squares. Students continue to build their academic vocabulary as they are asked to justify and explain their mathematical reasoning while answering questions. Throughout instruction, students review the lesson's vocabulary words and look up any word for which they do not know the meaning.





Models and Demonstrations

Research supports the use of models and demonstrations when teaching students new mathematical concepts and skills (Archer & Hughes, 2010; Gersten et al., 2008). Experts agree that these models and demonstrations should:

- Use precise wording to convey complex information, involve students in active demonstrations, and be selected carefully (Archer & Hughes, 2010; Doabler & Fien, 2013; Stein et al., 2006)
- Make transparent the thought processes that underlie the use of specific strategies (Rosenshine, 1995, p.267); One effective method for doing this is through think-alouds, in which teachers verbalize their thought processes as they solve problems (Archer & Hughes, 2011)
- Show different ways of approaching problems and include rationales for why one approach may be better than another, or when it doesn't matter (Sowder & Kelin, 1993)

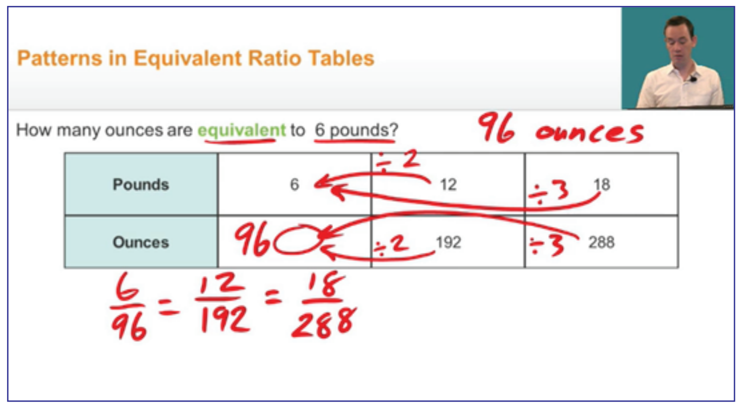
Worked examples that show worked-out solution steps for problems represent another effective form of modeling (Clark, 2005; Sweller, 2008). Research indicates that studying worked examples can elucidate why some solutions to problems are better than others and illuminate how to apply concepts in new settings (Sweller, 2008). As student expertise increases, fully worked examples should be replaced by partially worked examples, and eventually by problems students complete without support (Sweller, 2008).

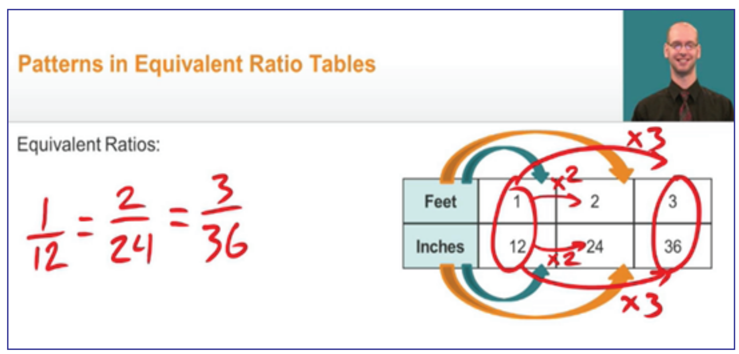
Students may also be taught "procedural prompts"—a specific set of steps or elements—to help them learn, remember, and apply cognitive strategies (Rosenshine, 1995, pp.266–267). For example, in learning how to generate questions about material they are reading, students may be taught to ask "who," "what," "why," and "when" questions. Specific types of graphic organizers may also function as procedural prompts, if students are taught to use those formats to organize their thinking and problem-solving.

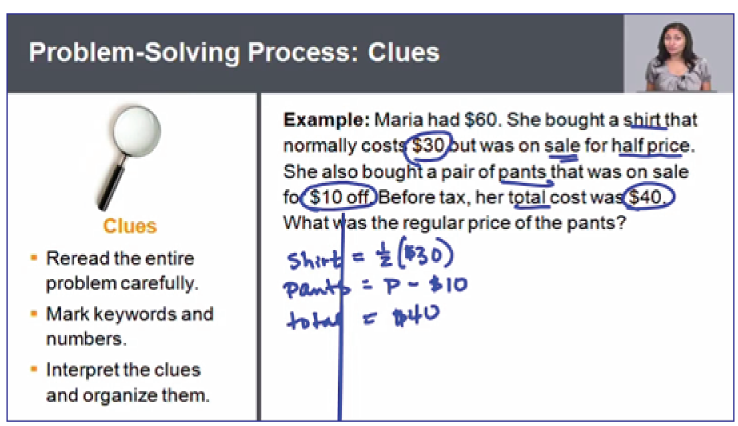
How Edgenuity Mathematics Courses Support Models and Demonstrations

Edgenuity mathematics courses capitalize on models, demonstrations, and worked examples to teach students critical concepts and procedures. Typically, students are provided with conceptual models and demonstrations before procedural ones.

On-screen teachers use clear-cut, consistent language to define terms and model concepts and procedures through think-alouds. For example, a grade 6 lesson teaches students how to find equivalent ratios using multiplication or division. The on-screen teacher first explains the conceptual underpinning of equivalent ratios. He reviews the concept that a ratio is used to relate two or more quantities. Then he explains that equivalent ratios show the same relationship. The on-screen instructor then reviews situations in which one would use equivalent ratios to solve problems. He illustrates







this by converting pounds to ounces, given a partially completed ratio table. He examines the relationship between 12 and 6 pounds, noting that 12 is twice as big as 6. Therefore, to determine how many ounces are equivalent to 6 pounds, he divides 192 ounces by 2. The on-screen teacher then models this procedure for converting inches to feet.

On-screen teachers also model problem-solving, cognitive, and metacognitive strategies. Students are provided with procedural prompts to support instruction in cognitive strategies. For example in mathematics courses, students are taught step-by-step methods (identify questions, clues, and strategies, and check their work) when solving problems. On-screen teachers think aloud while modeling this sequence of steps in the problem-solving process.

Additionally, on-screen teachers in Edgenuity mathematics courses regularly model thought processes that underlie the application of specific skills, procedures, and strategies. Think-aloud models in Edgenuity courses are clear, consistent, and concise, focusing on the critical aspects of the procedure. For example, in Algebra I, a think-aloud models the mental process of determining whether a graph is a function—with the on-screen teacher reminding himself that "in order for a graph to be a function, there needs to be one x for every unique y," and then checking to see whether this is true.

Engaging Tasks and Real-World Applications of Problems

Both domestic and international studies confirm that providing real-world applications of problems that stress students' understanding and application of mathematics improves mathematical achievement (Gersten et al., 2008; Ginsburg et al., 2005). Experts posit that presenting problems in real-world contexts can make mathematics more meaningful and accessible to students by helping them see the importance of what they are learning (Gersten et al., 2008). Research also suggests that both routine and non-routine applications in a wide variety of contexts can help students connect new knowledge to known knowledge, allowing "memory pathways [to] become more easily accessed and cross-referenced for future use" (McREL, 2010, p.100. See also Bransford, Brown, & Cocking, 2000; Ginsburg et al., 2005).

How Edgenuity Mathematics Courses Provide Engaging Tasks and Real-World Applications of Problems

Edgenuity's mathematics courses make learning relevant by linking instruction to real-world examples. A wide variety of activities enable students to transfer knowledge and skills to real-world situations. Students are asked to create, predict, explore, and write. For example:

- In a grade 6 lesson, students assume the role of a city planner and create an interactive model of a town to meet the constraints that 60 percent of the land is allocated to homes, 10 percent to shops, and 30 percent to parks.
- In a grade 7 lesson, students explore how to find the number of different species of fish in a pond by looking at random samples of fish. Using a simulator, students collect fish samples and predict the different types of fish in the larger population.
- In an Algebra II lesson, students evaluate the question, "Since the mid-1990s, women have earned more bachelor degrees than men. Does this mean that annual salaries have followed suit?" Using a national data set, students perform several regression analyses and predict the future relationship between salary and college degrees.

Scaffolded Practice

More than three decades of research and expert opinion support the use of scaffolded practice to improve mathematical performance (Archer & Hughes, 2011; Lajoie, 2005; NRC, 2012; Rosenshine & Stevens, 1986; Stein et al., 2006; Sweller, 2008). Practice (when a motor or cognitive skill is repeated over time) transfers knowledge from working memory to long-term memory. This allows for "accurate recall and applications in the future" (Sousa, 2008, p.64). Scaffolding refers to instructional support (i.e., prompts, cues, interactive tools, worked examples, graphic organizers, and other visual tools) that is gradually faded out as students complete tasks on their own (Stein et al., 2006). Experts agree that scaffolded practice is most effective when students receive immediate, corrective feedback to rectify erroneous thinking (Sousa, 2008).

How Edgenuity Mathematics Courses Support Scaffolded Practice

Edgenuity mathematics courses help students build mastery by providing scaffolded practice that is reduced over time. Each lesson carefully selects a small, meaningful amount of information for students to practice. On-screen instructors provide several examples and go over material in several different ways. During scaffolded practice:

- Cognitive, metacognitive, and comprehension prompts help students carry out their tasks. Visual and verbal cues are designed to focus students' attention and to help them check their own understanding.
- A dynamic glossary offers students a way to review the key academic vocabulary words from each lesson.
- Digital highlighters, sticky notes, calculators, graphic organizers, and other tools help students complete assignments.
- A digital notebook enables students to take notes, record observations, and synthesize information.
- "Show me" videos and hints clarify content.
- Multiple worked examples and models show how to complete tasks in a variety of ways. For example, on-screen teachers show students how an expert would think through a word problem using visual cues and verbal cues to identify important information.
- Immediate, corrective feedback reinforces correct performance and helps students make adjustments as needed. This is provided after every question during early supported practice, and after a question set is completed during later, more independent practice.

Customizing tools within Edgenuity's learning management system further scaffold learning. For example, teachers have the option to customize the course settings (e.g., adjust the time allotted for assessments and change the grade weights for activity types) to make the level of challenge more appropriate for individual students.

As students demonstrate understanding, on-screen teachers increase the complexity of tasks and decrease the level of prompts and support. For example, students may see partially worked examples instead of fully worked examples. Edgenuity courses also withdraw explanatory feedback as students demonstrate success.

Checks for Understanding and Immediate Feedback

Experts agree that checking for understanding is critical to improving students' mathematical performance (Archer & Hughes, 2011; Stein et al., 2006; Swan, 2003). To effectively check for understanding, students must receive "explanatory feedback that helps learners correct errors and practice correct procedures" (NRC, 2012, p.4-12). Such feedback can accelerate the rate of learning by eliminating the gap between students' conception of knowledge and the desired response (Archer & Hughes, 2011; NRC, 2012). Research indicates that online courses with multimedia are uniquely poised to not only provide students with immediate feedback on whether their answer is correct or incorrect, but also what can be done to improve future performance (NRC, 2012; Swan, 2003).

How Edgenuity Mathematics Courses Support Checks for Understanding and Immediate Feedback

Edgenuity offers three different types of checks for understanding:

- Formative assessments embedded within a lesson check understanding of concepts and skills as they are presented. Assignments, which follow the lesson, also serve as formative assessments. By providing corrective feedback, Edgenuity's formative assessments help students understand where their gaps in knowledge exist and learn where they need additional practice or support. After each instructional sequence, students answer questions and complete tasks to check what they learned and to apply new skills.
- Interim assessments (quizzes) occur after students finish an Edgenuity lesson. The items for these assessments are drawn from an item bank, each aligned to a specific lesson objective. Upon completion, students can view their performance by objective; this allows them to review areas of weakness before attempting the assessment again.
- Summative assessments (tests and cumulative exams) are provided at the end of each unit and/or course to evaluate students' overall performance.

Students receive immediate, appropriate feedback each time they respond to a question within an Edgenuity lesson.

- Feedback is provided with a human voice and is friendly, supportive, and explanatory.
- Feedback messages not only inform students of whether their responses are correct or incorrect, but also refine students'

understanding of concepts and to correct misconceptions.
- As noted above, feedback is provided after student practice—after every question during early supported practice and after a question set is completed for cumulative practice and review. Edgenuity courses withdraw explanatory feedback as students demonstrate success.

Review Over Time

Brain research suggests that learning is more effective when there are opportunities for "cognitive closure" and material can be summarized over time. During "cognitive closure," students "attach sense and meaning to new learning" and increase the chance that "knowledge will be retained in long-term memory" (Sousa, 2008, p.202).

As such, many educational researchers recommend that instructors "focus on prolonged, deliberate practice and application rather than one-shot deals" (NRC, 2012, p.6-24). Experts suggest that teachers provide two different types of practice over time:
- Massed practice that is repeated over a short period of time
- "Distributed practice" that is sustained over a longer period of time (Groth, 2013; Sousa, 2008)

How Edgenuity Mathematics Courses Support Review Over Time

Students review skills and content at the end of every topic/unit, as well as at the end of each semester. Edgenuity test and exam reviews represent opportunities for cumulative practice that integrates previously learned skills in a meaningful context. Because review activities are generated dynamically based on the course content students actually completed, they are automatically designed to provide cumulative practice of only the skills each student actually covered in instruction.

Deep Content Learning

Educational and cognitive researchers alike posit that "deep understanding" of content learning occurs when concepts (ideas or mental representations) are well represented and connected (NRC, 2012; Sousa, 2008; Zirbel, 2006). Students who exhibit deep understanding are not only able to make sense of a singular concept, but are also able to logically organize, explain, and make connections across multiple complex topics.

To build deep understanding, experts recommend engaging students in higher-order processes such as understanding, application, analysis, evaluation, and creation (Mayer, 2002; Karthwohl, 2002). They also suggest focusing instruction on a few big ideas that connect and organize mathematical concepts (Schmidt et. al., 2002; Charles, 2005; NCTM, 2008; National Governor's Association & Council of Chief State School Officers, 2010). As Charles (2005, p.10) notes, "[W]hen one understands big ideas, mathematics is no longer seen as a set of disconnected concepts, skills, and facts. Rather, mathematics becomes a coherent set of ideas."

This section shows how Edgenuity mathematics courses develop deep thinking in four discrete areas:
- Numbers and operations
- Algebra and functions
- Statistics and probability
- Geometry

Number Sense

One of the primary goals of middle and high school mathematics instruction is to build students' capacity to look for patterns, make conjectures, visualize quantities, and solve problems (Groth, 2013). This capacity depends on students having solid number sense, or the ability to recognize number meaning, number relationships, number representations, number magnitude, and number reasonableness (Yang & Tsai, 2010; NCTM, 2008). Students with strong number sense are able to transform numbers or alter the mathematical structure of problems to make them more manageable (Groth, 2013; Sowder, 1992). They also understand the impact of operations on numbers. To boost number sense in middle and high school students, educational researchers recommend providing opportunities for students to practice mental and computational estimation (Groth, 2013; Owens, 1993; Sowder & Kelin, 1993).

How Edgenuity Mathematics Courses Foster Number Sense

Edgenuity recognizes the importance of helping students develop flexible thinking about numbers. Throughout lessons, on-screen teachers model mental and computational estimation. For example:

- In a grade 6 lesson, an on-screen teacher models how to estimate decimal products. Before he shows how to calculate 37.8×11.2, he makes an estimate about the product. The teacher models his reasoning by saying he wants to find numbers that are easier to multiply in his head. He notes that 37.8 is close to 40 and that 11.12 is close to 10, so the product is close to $40 \times 10 = 400$. He then uses the distributive property to find the actual product, 423.36. When analyzing the number, the on-screen teacher describes how the exact product will be greater than or less than the estimate, depending on how the factors were rounded.
- In a grade 8 lesson, an on-screen teacher shows how to estimate and compare roots by looking at the nearest whole number square, the nearest tenth place, and the nearest hundredth place.

Edgenuity offers lessons that focus student attention on number size, number magnitude, and number reasonableness. For example:

- In a grade 6 lesson on comparing ratios and rational numbers, the on-screen teacher emphasizes the meaning behind fraction symbols. The teacher stresses that one has to look at both the numerator and the denominator when ordering and comparing fractions. She explains why $\frac{5}{6}$ is smaller than $\frac{5}{9}$.
- In a grade 8 lesson on scientific notation, the on-screen teacher stresses that students cannot compare decimal parts alone and that "more digits do not make a number bigger." He demonstrates why 0.1814 is not the largest number out of 0.09, 0.385, 0.3, and 0.1814.
- In a grade 7 lesson on estimating percentages, students are asked to determine whether $40 is a reasonable amount of money to pay for a cab fare that costs $32 with a 20 percent gratuity.

Proportional Reasoning Skills

Proportional reasoning involves the deliberate use of multiplication to compare quantities and predict the value of one quantity based on the value of another. Many researchers regard proportional reasoning as the bedrock of mathematical learning, due to its prominent role in numeration, measurement, geometry, algebra, and probability (Groth, 2013). As Lamon (2007, p.629) points out, proportional reasoning is "the most protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, but also the most essential to success in higher mathematics."

Studies reveal that students struggle with proportional reasoning for a variety of reasons. Many students fail to:

- Understand that the numerator and denominator of fractions cannot be operated on as separate entities (Beher et al., 1984; Mack, 1990)
- Recognize that fractions can have multiple meanings—part/whole, decimals, ratios, quotients, measures, or operators (Watson et al., 2013)
- Comprehend the concept of fraction equivalence, proportionality, and relating fractions to division (Kerslake, 1986)
- Use multiplicative strategies when solving proportional problems (Watson et al., 2013)
- Understand how quantities relate, co-vary, or change together; that is, they do not understand that sometimes part of the quantity relationship is proportional and other times it is constant (Langrall & Swafford, 2010)
- Grasp the difference between linear and non-linear situations (Watson et al., 2013)

Fortunately, research has uncovered several instructional strategies that can bolster students' proportional reasoning skills. They include:

- Focusing on the big idea that proportional reasoning involves multiplicative comparison (Carnine, Jones & Dixon, 1994; Siegler et al., 2010; Van de Walle, 2006; Watson et al., 2013)
- Illustrating how fractions can be interpreted and represented in a variety of ways (Groth, 2013; Kieran, 1992; Lamon, 2005; Watson et al., 2013)
- Developing conceptual understanding before formal vocabulary and procedures are introduced (Groth, 2013)
- Identifying situations where ratios and proportions are appropriate to solve problems (Langrall & Swafford, 2000)
- Teaching students to "recognize the difference between absolute, or additive, and relative or multiplicative change" (Langrall & Swafford, 2000, p.255)
- Distinguishing between proportional and non-proportional relationships (Watson et al., 2013)
- Capitalizing on a wide variety of visual representations (Cramer, Wyberg, & Leavitt, 2008; Siegler et al., 2010; Watson et al., 2013)

How Edgenuity Mathematics Courses Foster Proportional Reasoning Skills

Edgenuity mathematics courses emphasize the development of proportional reasoning skills. Throughout mathematics lessons, students are reminded that ratios compare two or more quantities and that fractions compare parts to whole quantities. On-screen teachers demonstrate how fractions can be expressed in different ways. For example, a grade 6 lesson points out that the relationship between the quantities one apple and two pears can be written as 1:2, ½, or "one to two."

Lessons illustrate how fractions can be used to represent different ideas in different contexts. They show:

- Part-to-whole interpretations, where ¾ can represent 3 slices out of 4 equal size pizza slices
- Measurement interpretations, where ¾ can represent 3 lengths of size that are ¼ units each
- Operator interpretations, where ¾ can represent 75 percent of an image
- Ratio interpretations, where ¾ can be described as a ratio (there are 3 parts lemonade for every 4 parts strawberries)
- Quotient interpretations, where ¾ of a cup of dog food can represent 3 cups divided equally among 4 puppies

Courses deliberately build conceptual understanding before teaching students vocabulary and procedures. For example, a grade 6 lesson first asks how much greater ⅞ is than ¾, thereby laying the groundwork for common denominators and adding and subtracting factions. Students gain experience with this concept before the term "common denominator" is formally defined and explained.

On-screen instructors stress solving ratio problems using a multiplicative, not additive approach. They clarify when multiplication can be used to solve problems. In a grade 6 lesson, the on-screen teacher explains that multiplication can be used to convert larger units to smaller units of measurement, because it takes more of the smaller unit to make up the distance of the larger unit. Conversely, the on-screen teacher shows how division can be used to convert smaller units to larger units of measurement.

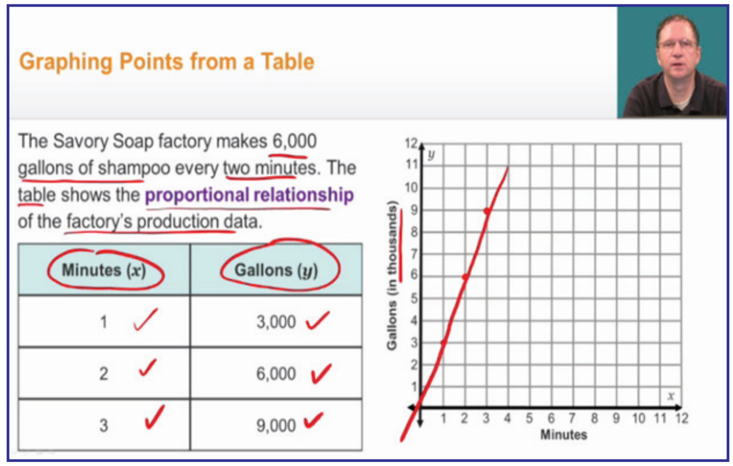
In Algebra I, the on-screen teacher discusses how the goal of solving linear equations is to figure out an unknown. To isolate a variable, students are taught to use the multiplication property of equality. In Algebra II, the on-screen teacher illustrates how multiplication can be used to measure the way one variable changes in relation to another.

Students are encouraged to set up proportions in tables, tape diagrams, graphs, and equations that highlight the multiplicative relationships between two quantities. Students learn when and why additive approaches can be less efficient than multiplicative ones.

On-screen instructors also model how ratios can be used to solve constant-rate word problems. They provide examples of how to use proportions to calculate unit rates, make conversions, identify percentages and percentage of increase or decrease, determine changing units of measurement, predict the probability of an event, compare two amounts, pinpoint scale and size, and identify a part of the ratio when one knows another part.

They also identify linear and non-linear relationships. For example, in grade 7 lessons, students use ratio tables to explore proportional relationships. As they translate rows of the table into ordered pairs and graph them in the coordinate plane, students visualize a proportional relationship as a straight line that passes through the origin.

When possible, mathematics lessons encourage students to redefine the fundamental unit with which they are working. For example, in a grade 6 lesson, if you want to know the cost of 24 balloons, given that 3 balloons cost $2, it is easier to reason in terms of three-packs of balloons rather than in terms of a single balloon, since 3 divides 24.



Area models, fraction bars, graphs, and grids help students apply ratios to real-world problems involving recipes, conversions, speed, scaling, mixture, rates, density, percents, scale drawings, probability, rates of change, geometry, and trigonometry.

Algebraic Thinking

Algebra is the branch of mathematics that uses tables, graphs, and symbols to express patterns, relations, and functions in real-world contexts. According to Driscoll (1999), strong algebraic thinking revolves around "certain habits of mind": building rules to represent functions, executing procedures and doing them in reverse, and abstracting from computation. According to Watson et al. (2013, p.16), this entails "formulating, transforming, and understanding generalizations of numerical and spatial situations and relations" as well as "using symbolic models to predict and explain mathematical and other situations."

This section describes how Edgenuity helps students interpret literal symbols, understand algebraic structure, operations, and procedures, and make sense of patterns.

Interpreting Literal Symbols

Research supports direct instruction on how to interpret literal symbols (Groth, 2013; Kieran, 1992; Watson, 2007; Welder, 2012). Experts agree that generalizing from a literal symbol, or letter of the alphabet, is the core of algebra (Watson, 2007). More than 30 years of research, however, shows that students struggle to understand the meanings of letters and how to use them (Booth, 1984; Groth, 2013; Kieran, 1992; Kuchemann, 1981; MacGregor & Stacey, 1997; Watson, 2007; Welder, 2012). Students fail to recognize that symbols can represent unknowns, labels, placeholders, variables, or constants (Groth, 2013, Watson, 2007). To successfully develop algebraic reasoning skills, students must learn "that there are different uses for different letters in mathematical conventions" (Watson, 2007, p.13), and see the differences between them.

Research also underscores the importance of teaching students to translate between words and algebraic symbols (Groth, 2013). Studies (Groth, 2013; MacGregor & Stacey, 1997; Welder, 2012) have documented that students make reversal errors of symbols when translating word problems to algebraic symbols. In the well-cited "student-professor problem," researchers note that students often mistakenly interpret the statement, "There are six times as many students as professors at this university" as 6S = P. They don't understand that this equation implies the exact opposite of what they want to convey: that there are six times as many professors as there are students. Many students believe algebraic symbols in an equation should be written in the same order in which they occurred in the word problem. Effective instruction should illustrate how the structure of a sentence can differ from the structure of an algebraic equation (Groth, 2013).

How Edgenuity Mathematics Courses Teach Students About Literal Symbols

A prominent theme throughout Edgenuity mathematics courses is that the role of literal symbols—from labels, givens, unknowns, parameters, and constants—changes depending on context. On-screen instructors explicitly stress that algebra can be used to create meaningful expressions and equations that represent the real world.

In grade 6 lessons, students begin by examining specific unknowns when exploring numerical expressions to express quantities. First, they see how a literal symbol can be evaluated and mentally replaced with a number ($a + 5 = 8$). They also explore how computations can be performed around a literal symbol. For example, to solve $44 = 7x + 2$, one "ignores" the literal symbol, subtracts 2, and divides each side by 7. Here, the on-screen instructor points out that the unknown represents one number that can be evaluated and replaced with another number. During this time, on-screen instructors show how this method works to solve equations such as $ax + b = c$, but cannot be used to solve equations such as $ax + b = cx + d$, because the literal symbols are on both sides.

Students extend that understanding to simple algebraic expressions—using literal symbols to represent numbers that can change and have multiple values. For example, students learn that literal symbols can have a relationship with another letter (e.g., $y = x + 2$ represents a quantity that is 2 more than whatever x is, or $y = 2x$ where y is twice as big as x).

On-screen instructors point out the role of literal symbols in formulas, as they signify numbers in generalizable arithmetic. For example, in the formula for the area of a triangle ($A = bh$), the literal symbols b and h stand in for the generalized rule: Multiply whatever the base is by whatever the height is.

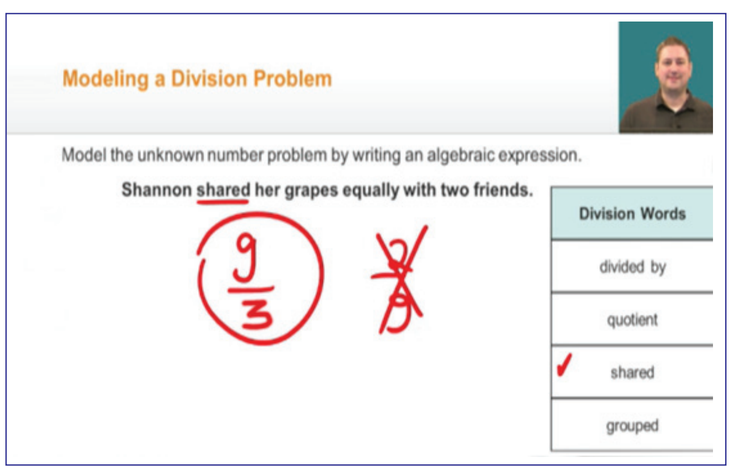
Grade 6 lessons also show that literal symbols can be manipulated as discrete objects when solving equations by combining similar terms (e.g., $2x + 3x = 5x$). Here, the on-screen teacher explains that the letter x represents a specific unknown number and does not have to be considered consciously to manipulate the symbols.

In addition, students explore the role of literal symbols in equivalent expressions, evaluating whether two expressions are equivalent all the time, some of the time, or never. They simplify and evaluate algebraic expressions. This forms a foundation for the study of equations—in which two expressions are defined as equal.

Finally, 6th grade students explore the role of literal symbols in an inequality—where the variable represents a range of values, not just a single number. They graph solutions to inequalities on number lines to reinforce the concept that the solution is a range of numbers.

Students also see that a variable can represent a constant specifying the slope or y-intercept of a line. When studying the expression $y = mx + b$, students see that b and m represent constants specific to the equation, and that x is the independent and y is the dependent variable.

In grade 7 lessons, students extend their understanding of expressions, equations, and inequalities to more complex examples with multiple steps. Students review and reinforce the many definitions of variables in different algebraic contexts. On-screen instructors show that different literal symbols do not always equate to different solutions. For example, the literal symbols m and x in the equations $3m + 4 = 9$ and $3x + 4 = 9$ represent the same value.



Grade 8 students learn that when a literal symbol is used as a generalized number, it represents the values of a set of numbers rather than a single value, such as in the property $1 = n * (1/n)$. They also formally study functions, defining one literal symbol in terms of another in the exploration of bivariate data. Students compare and contrast the role of variables in parabolas, binomials, trinomials, and rational expressions.

Edgenuity mathematics courses also teach students to think about the operations needed to translate between words and algebraic symbols. Students are taught to look for key clues (e.g., "quotient" indicates the result of a division problem), identify what variables are known and unknown, and then determine which operation needs to take place.

For example, in a grade 6 lesson, the on-screen teacher models how to translate the phrase, "Shannon shared her grapes equally with two friends" into an expression. First, the on-screen teacher states that since the value of grapes is an unknown, it can be represented as "g." Next, the he reminds students to think about the operation that needs to take place: Based on Shannon's "sharing grapes equally" with her two friends, the teacher knows he needs to divide that number of grapes by 3. Shannon sharing with her friends implies that she will also get grapes, so the division should be by 3, not 2. So the expression g/3 represents the number of grapes each person will get. The instructor is careful to point out why 3/g does not accurately represent the number of grapes each person will get, because we don't want to divide 3 by the number of grapes.

On-screen teachers also coach students on how to contextualize algebraic expressions and equations. For example, in a grade 7 lesson, the on-screen instructor illustrates how 13.47 – y means "the difference of 13.47 and a number." The on-screen teacher points out that the expression cannot be "13.47 subtracted from a number" because y is being taken away, not 13.47.

Understanding Algebraic Structure and Relations

To succeed in algebra, students need to have a relational view of the equal sign; that is, they need to understand that = means "equal to" rather than "makes" or "to do." Students who have a relational view of the equal sign tend to score higher on standardized mathematics test and are less likely to use ineffective guess-and-check strategies to solve problems (Knuth, Stephens, McNeil, & Alibali, 2006). Effective instruction must address the meaning of the equal sign so that "arithmetic becomes the study of relations among numbers rather than purely about computation" (Watson et al., 2013, p.23).
Students must also have a deep understanding of the relations that underlie algebra operations (Driscoll, 1999; Watson, 2013). Effective instruction needs to help students move between procedural and structural conceptions of problems (Kieran, 1992). In reviewing decades of research on algebra learning and instruction, Watson (2007, p.23) found that students "tend to use procedural manipulations when solving equations and inequalities without a mental image or understanding to prevent errors." Similarly, Groth (2013, p.218) points out that learners tend "to develop their own sets of beliefs about how operations should be carried out." To build algebraic reasoning skills, students need to not only understand "the nature of multiplication and division—both as the inverse of multiplication and as the structure of fractions," but also that "subtraction and division are non-commutative, and that their inverses are not necessarily addition and multiplication" (p.18).

How Edgenuity Teaches Students to Think About the Structure of Operations and Their Relations

Designed to teach students about algebraic structure and relations, Edgenuity's mathematics courses promote a relational view of the equal sign. On-screen instructors discuss the meaning of the equal sign frequently throughout lessons and explicitly model correct use of the equal sign. Students are reminded that the equal sign means that two quantities have the same quantitative value. For example:

- In a grade 6 lesson, an on-screen instructor uses an area model to demonstrate equivalence.
- In a grade 8 lesson, students gain a relational understanding of the equal sign by translating it into word problems and answering questions about equations with fractions.
- In an Algebra II lesson, the on-screen teacher models the concept of equivalence with pan-balance scales, showing that when the two sides have the same weight, they are equivalent.

Mathematics courses also stress the underlying additive and multiplicative relations behind operations. For example, on-screen teachers remind students that:

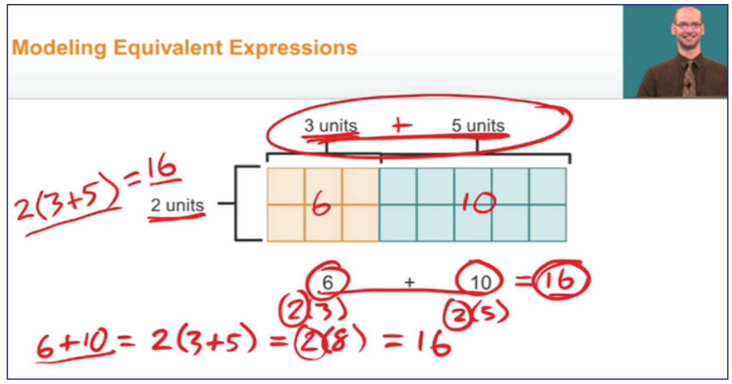
- Equations such as a + b = c (e.g., 2 + 3 = 5) are additive. Because addition and subtraction have an inverse relationship, one undoes the other. This means one can find x if 2 + x = 5.

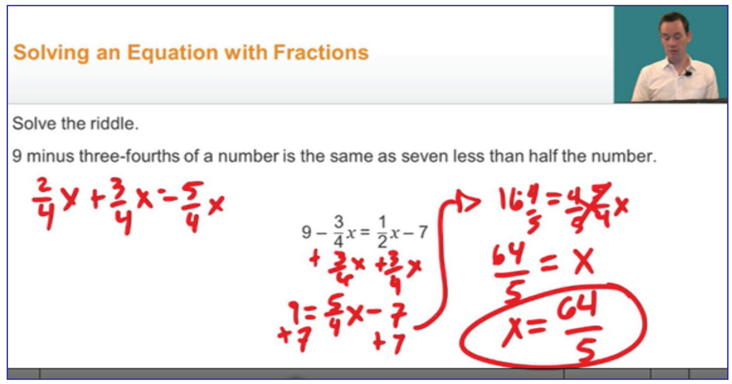
- Equations such as c = ba are multiplicative. Because multiplication and division have an inverse relationship, this means a = c/b and c/a = b.
- Equations such as (a + b) + c = a + (b + c) and (ab)c = a(bc) are associative, meaning it doesn't matter how you group the numbers or which numbers you calculate first. This does not work for subtraction.
- Expressions such as a(b + c) can be distributed, so that the equation equals ab + ac. For example, 3a + 3b means three of quantity a added to three of quantity b can be expressed as 3a + 3b or 3(a +b); they are equivalent.
- Addition and multiplication are both commutative. Subtraction and division are not.

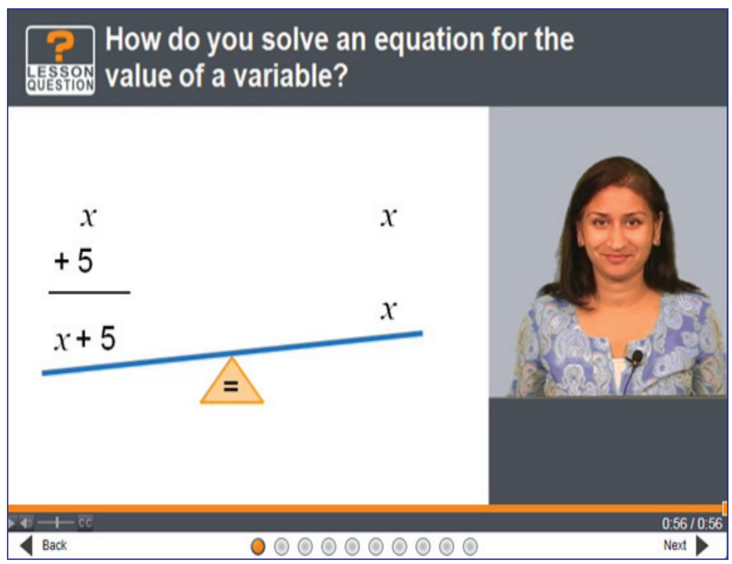
On-screen teachers directly address common misconceptions about relations that underlie operations. For example:

- Students solve inequalities using algebraic symbol manipulation, number lines, and function graphs. In doing so, they are reminded that division by a negative number changes the direction of the inequality.
- In a grade 7 lesson, an on-screen instructor explains how $(24)^3$ is conceptually the same as 24*24*24 and that is the same as 2*2*2*2 * 2*2*2*2 * 2*2*2*2, which is the same as 2^{12}. After identifying the relations behind the structures, students learn that a shortcut for multiplying the number 2 twelve times is to multiply the exponents.
- Using algebra tiles, students learn that the area of a rectangle with dimensions (x + 1) and (x + 3) is x^2 + x + x + x + x + 1 + 1 + 1, or x^2 + 4x + 3.

Students are given multiple opportunities to apply different operations to a wide array of formulas, equations, identities, properties, and functions. Tasks and assignments involving words, tables, graphs, and symbols allow students to see how different operations can affect expressions, inequalities, equalities, and functions.



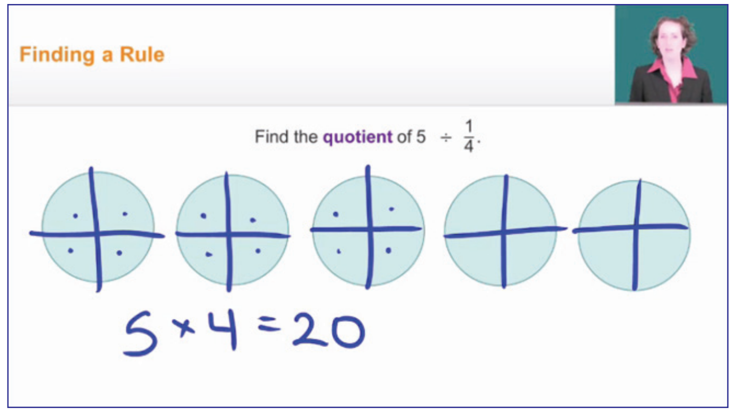




Abstracting from Computations and Generalizing from Patterns

An important goal of algebra is to help students abstract from computations. As Driscoll (1999, p.2) points out, "[T]hinking algebraically involves being able to think about computations freed from the particular numbers they are tied to in arithmetic." This entails using shortcuts and being flexible with numbers to ease mental calculations. For example, a student who is able to think about computations independently will recognize "that 101 can be decomposed into 100 + 1; 99 + 2; 98 + 3; and so on" (Driscoll, 1999, p.2).

One way to help students abstract from computations is to examine patterns and make generalizations. Encouraging students to compare representations of relationships in graphical, numerical, symbolic, and tabular form allows for diverse thinking strategies. As Watson (2007, p.28) points out, "[L]earners who have combined pattern-generalizations... and other ways to see relationships can become more fluent in expressing generalities in unfamiliar situations."



How Edgenuity Mathematics Courses Teach Students to Abstract from Computations and Generalize from Patterns

Edgenuity on-screen instructors helps students abstract from computations. For example, in a grade 6 lesson, an on-screen teacher models how a student can use a shortcut to find the quotient of 5 divided by ¼. Using an area model, the instructor points out that she could count each fourth to find the quotient, but the faster way to solve the problem would be to count groups of four by multiplying by 5.

In Edgenuity courses, students are required to complete a number of tasks and assignments that ask them to generalize from patterns. For example:

- In an Algebra II lesson, students analyze numeric, geometric, and algebraic patterns to determine whether a function could be used to model a rope swing.
- In a grade 8 lesson, a performance task requires students to analyze how the number of customers served in a restaurant relates to the amount of tips collected per day. Students organize data tables, create scatterplots, and fit trend lines, to explore and generalize about patterns in the data.

Spatial and Geometric Thinking

Effective geometry instruction involves teaching students spatial thinking and deductive reasoning. Spatial reasoning "consists of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed," (Clements & Battista, 1992, p.420). Deductive reasoning "involves incorporating accepted statements such as theorems, postulates, and definitions into a logical argument" (Groth, 2013, p.318).

Dina and Pierre van Hiele's research indicate that spatial and deductive reasoning develops in five distinct phases (see Clements & Battista,1992).

- Level 1: Visual—students identify geometric configurations (rhombus, squares), but they cannot identify the shape's properties.
- Level 2: Descriptive/Analytical—students can characterize shapes by their properties, but only through empirical tests such as measuring. They cannot see relationships between shapes.
- Level 3: Abstract/Rational—students understand the relationships between shapes, but they cannot use deduction to form geometric truths.
- Level 4: Deductive—students can establish theorems and know the difference between undefined terms, definitions, axioms, and theorems in deductive reasoning.
- Level 5: Rigor—students can reason by manipulating geometric statements, definitions, and theorems.

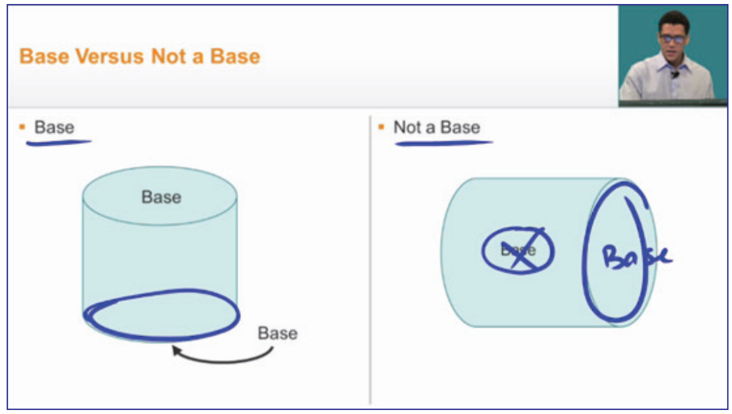
According to the Van Hiele model, the goal of middle and high school course curricula is to achieve levels 3 and 4, respectively. However, a national study conducted by Usiskin (1982) found that the large majority of students do not reach level 2.

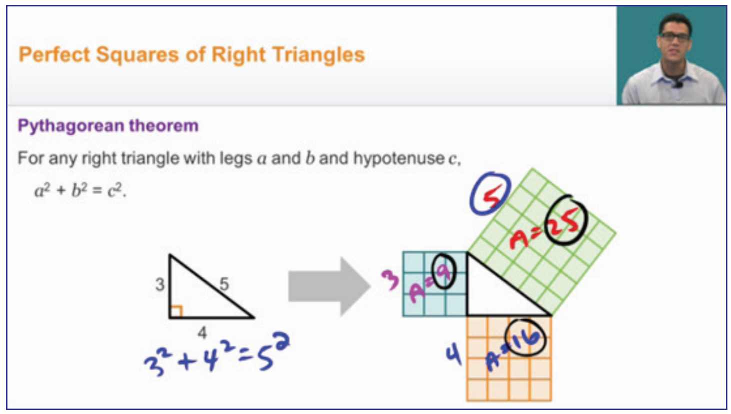
Empirical data show that many students have deeply ingrained misconceptions about the definitions of shapes and measurements that impede their geometric learning (Clements & Battisa, 1992; Groth, 2013; Watson et al., 2013). All too often, instruction focuses "on recognition of shapes and development of vocabulary rather than on development of concepts" (Geddes & Fortunato,1993, p.204). As a result, students don't get the rich conceptual understanding they need to develop strong spatial reasoning skills.

To improve spatial reasoning, experts (Clements & Battista, 1992; Geddes & Fortunato,1993; Groth, 2013) suggest that instruction should:

- Provide a wide variety of exemplars
- Carefully draw distinctions between common usage of definitions and mathematical usage
- Explicitly discuss the meanings and concepts behind geometric diagrams
- Provide opportunities for students to use technology to gain conceptual understanding

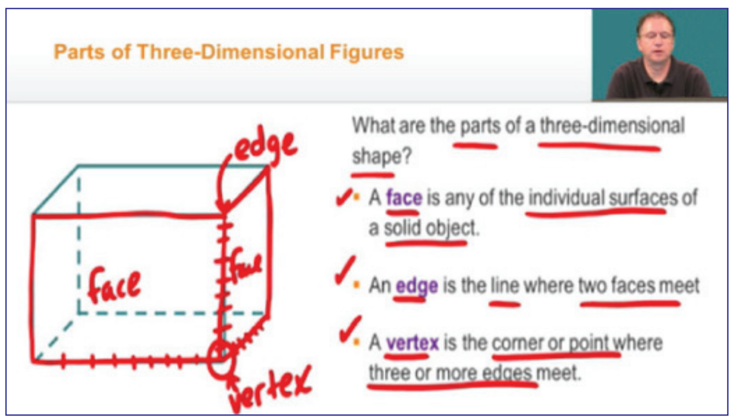
In addition, geometry instruction should encourage students to conjecture, build chains of reasoning, and validate arguments to strengthen their deductive reasoning skills (Clements and Battista, 1992, p.442; Groth, 2013, p.319). Finally, instruction should provide real-world applications of material.



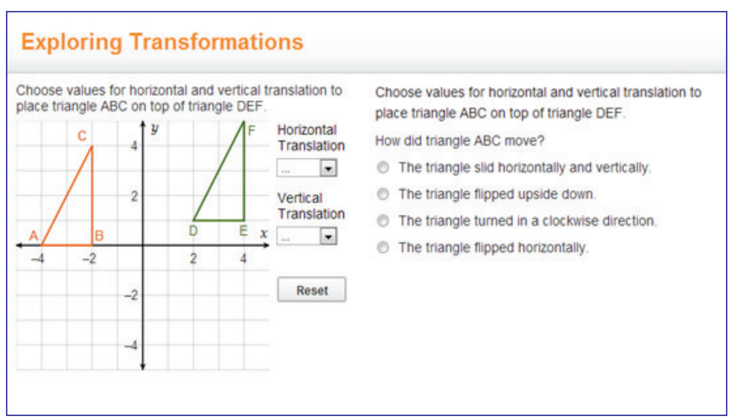


How Edgenuity Mathematics Courses Foster Geometric Thinking Skills

Edgenuity recognizes the importance of clear instruction. Definitions are unambiguous and on-screen instructors provide a wide array of examples and non-examples. Diagrams are clearly labeled and specifically call out key visual elements to build spatial reasoning.



On-screen instructors directly confront common misconceptions. For example, they illustrate why the area of a quadrilateral cannot be obtained by transforming it into a rectangle with the same perimeter. They prompt students to think about the critical characteristics of shapes.



Assignments and tasks ask students to describe the properties behind geometric figures in detailed diagrams and identify and compare shapes' properties. For example, in a grade 7 lesson, the on-screen instructor compares the volume of a rectangular pyramid to that of a rectangular prism with the same base.

Careful efforts are made to help students see the relationships between objects. For example, in a grade 8 lesson that introduces the Pythagorean theorem, the on-screen instructor uses an area model to demonstrate that the sum of the areas of the squares constructed on the legs of a right triangle will equal the area of the square constructed on the hypotenuse.

In a grade 6 lesson, Edgenuity asks students to determine the number of cubes it would take to fill different boxes, represented by two-dimensional drawings. Students make conjectures, compare them, and then test the conjectures using electronic cubes and boxes. Discrepancies between predicted and actual results prompt students to go back and revise their thinking. Throughout courses, students are taught problem-solving strategies for how to use deductive reasoning to create theorems, using both definitions and undefined terms.

Using interactive tools and simulations, students gain a deeper understanding of the material. For example, grade 8 students investigate triangular prisms. They also learn how translations, rotations, and reflections preserve the distance between points. Assignments build deductive reasoning by having students explain why the product of rotations is either a rotation or a translation.

Throughout the high school geometry course, students learn how concepts can be used to model and solve real-world problems. Students apply their knowledge of symmetry to make scale models of buildings, geometric vectors to model the velocity of kayakers in water, and trigonometry and angles of elevation and depression to find the distance between objects (e.g., a lighthouse and boat in the water).

Statistical Reasoning

Statistical reasoning is “the way people reason with statistical ideas and make sense of information” (Garfield, 2002, p.1). According to Garfield, this not only involves making “interpretations based on sets of data, graphical representations, and statistical summaries” but also includes “understanding distribution, center, spread, association, uncertainty, randomness, and sampling” (Garfield, 2002, p.1). Over the past 20 years, there has been a growing body of research on how to improve students’ statistical reasoning skills (Groth, 2013).

Research has documented a number of difficulties students face when learning probability and statistics. Many students struggle with interpreting graphs and analyzing summary statistics (mean, median, mode, spread, interquartile range) about data sets (Groth, 2013; Shaughnessy, 1992). Bryant and Nunes (2012) documented a number of prevalent probability misconceptions. Their review of the research noted that students:

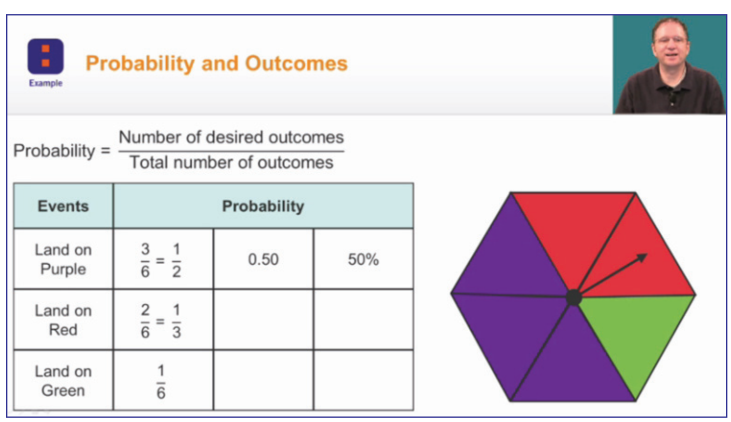
- Often mistakenly believe after a run of one kind of outcome that a different outcome is more likely the next time
- Don’t properly analyze or classify the sample space
- Have difficulty applying proportional reasoning to probability
- Have difficulty understanding how sample size is related to probability
- Have difficulty understanding conditional probabilities

Studies show that these difficulties can be addressed by helping students conduct statistical investigations, use technology, and run probability simulations (Groth, 2013; Shaughnessy, 1993). To maximize probabilistic thinking, instruction should expose students to experimental probability, theoretical probability, and subjective probability.

How Edgenuity Mathematics Courses Foster Statistical and Probabilistic Thinking Skills

Throughout Edgenuity lessons, students investigate concepts of sample space by comparing and contrasting geometric probability, theoretical probability, and experimental probability. Students also investigate probability with compound events.

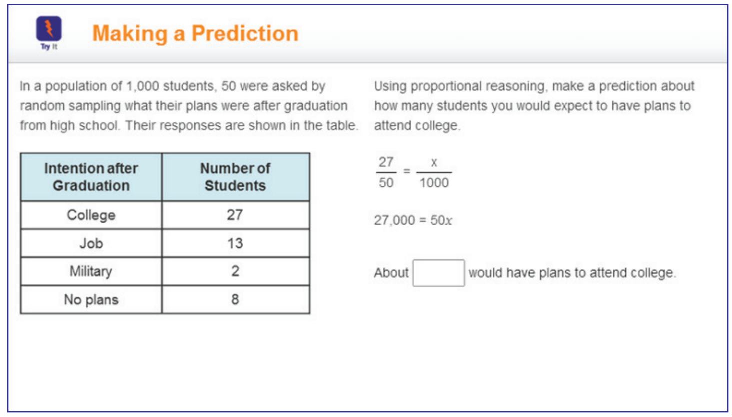
On-screen teachers provide conceptual definitions and directly address misconceptions. For example, in a grade 7 lesson, when learning to calculate outcomes for an event, students use tables, tree diagrams, organized lists, and the fundamental counting principle. After exposing students to different strategies, students report back on the approach that was most effective for them in solving problems. In another lesson, on-screen instructors demonstrate how to use sampling to make predictions about populations.

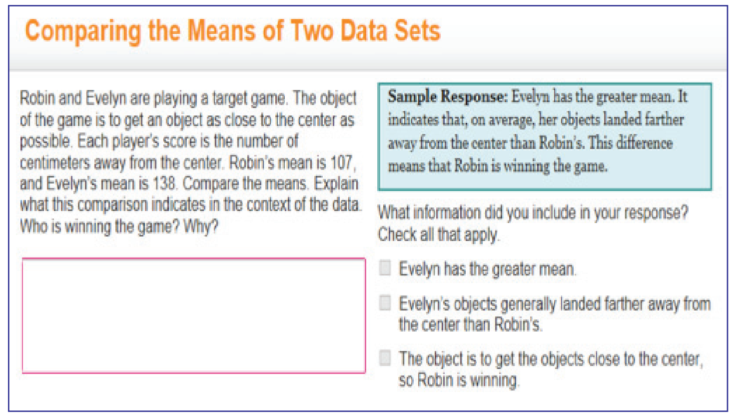


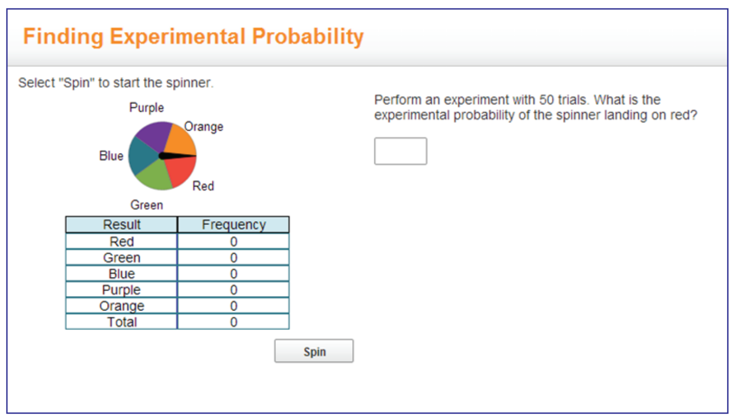
Through interactive assignments, not only do students compare the median, mean, distance from the mean, and mean absolute deviation, but they also compare measures of center and variability. This provides students with a deeper ability to describe statistics.

Learners are provided with multiple opportunities to analyze data trends by analyzing, graphing, and comparing dot plots and box plots. Students also get practice with random sampling and multiple samples to make inferences about populations. Edgenuity teaches students to identify and graph factors that are relevant, constructing multiple statistical representations of a system and communicating to others what the statistical system suggests. By avoiding tasks that require students to produce graphs as ends in themselves and encouraging more meaningful activities, on-screen educators help build students' graph comprehension.

Edgenuity capitalizes on simulations to help students experiment and explore the sample space. For example, in a grade 7 lesson, students use a virtual spinner to find experimental probability. Because students are able to perform experiments multiple times, they are able to explore the law of large numbers—which states that the larger the sample, the more likely the result is a good prediction for the whole population. In addition, students create their own sample spaces and make predictions about theoretical probabilities of certain outcomes. They then compare their prediction with the actual results.







Mathematical Practices

Standards for Mathematical Practice

The Common Core State Standards (National Governor's Association for Best Practices & Council of Chief State Officers, 2010) include eight Standards for Mathematical Practice that articulate the skills and habits of mind needed to successfully learn mathematics. In many ways, the eight standards build from the National Council of Teachers of Mathematics process standards (communication, representation, reasoning and proof of connections, and problem solving) and the five strands of mathematical proficiency (procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition) articulated in "The National Research Council—Adding It Up." The Standards for Mathematical Practice are:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Edgenuity's courses fully integrate the Standards for Mathematical Practice.

How Edgenuity Mathematics Courses Teach Students to Persevere While Solving Problems

Throughout Edgenuity's mathematics courses, students are taught and given ample opportunities to solve word problems. On-screen teachers explicitly model problem-solving and metacognitive strategies. Students are taught a multi-step process for solving problems. They learn how to use mnemonics, graphic organizers, checklists, and problem-solving strategies to check, process, and retrieve information.

To help students not only make sense of problems, but also persevere in solving them, on-screen instructors prompt students to think strategically by asking questions such as:

- What is the question at hand?
- What words or ideas cue the information needed to solve the problem?
- What strategies and data are needed to solve the problem?
- How do my prior experiences relate to the problem?
- What can I draw or write down to solve the problem? Can I make a chart? A list? A diagram?
- What do I think the outcome will be?

On-screen instructors in Edgenuity mathematics courses also prompt students to highlight, underline, and take notes to determine the importance of information. The instructors use think-aloud demonstrations to model their thinking and provide guidance on what students should be alert for as they annotate. For example, in an Algebra II course, an on-screen instructor highlights the important words needed to translate a word problem on alkalinity into a logarithmic equation.

Students then work on assignments and tasks independently, and explain their reasoning and thought processes. Through this process, students sharpen their reasoning and problem-solving skills.

How Edgenuity Mathematics Courses Teach Students to Reason Abstractly

On-screen teachers help students build their capacity to reason abstractly and quantitatively. They make explicit connections between mathematical phenomena and the real world, and explain how symbols represent objects. In middle school courses, on-screen teachers help students think about whether a fraction represents a part-to-part relationship or a part-to-whole relationship. They also guide students in how to find patterns and generalize from them to solve a problem. The on-screen teachers model how to compare and contrast different problem-solving strategies. Responsibility is transferred gradually, until students reason abstractly on their own.

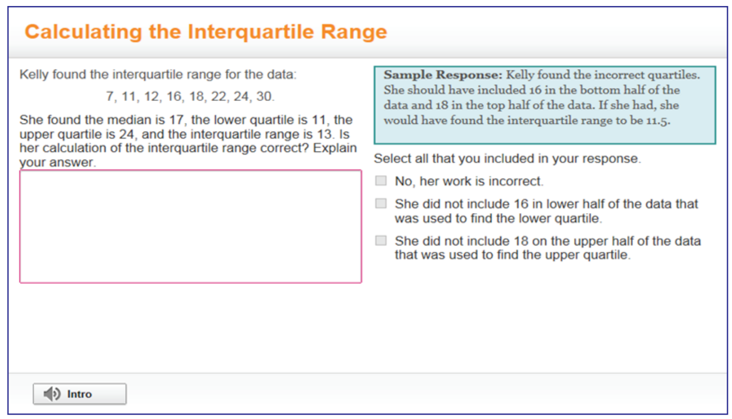
Students in Edgenuity mathematics courses are taught to monitor their understanding of word problems by asking themselves questions such as:

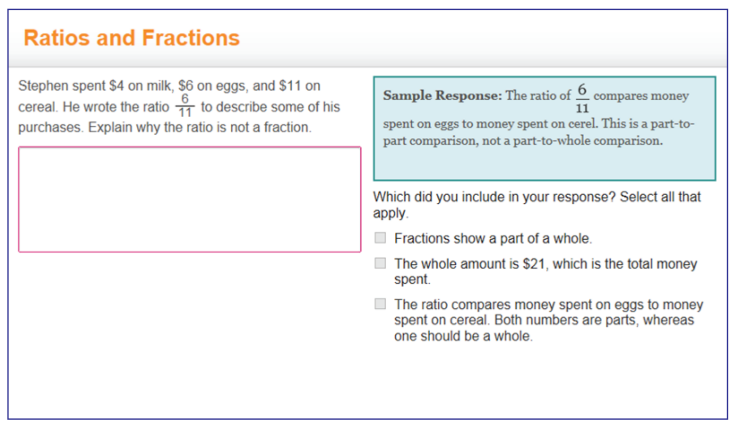
- Are there words I don't know that I must understand to solve the problem?
- Will the strategy I'm using to solve this problem be helpful?
- Am I learning anything important as I solve the problem?
- Am I making mistakes?
- Do I need to revise my strategy?

How Edgenuity Mathematics Courses Teach Students to Construct Viable Arguments

Edgenuity's assignments give students multiple opportunities to present, justify, and defend their solutions and methods. Students analyze the answers of others by identifying errors in the work. In addition, students construct arguments to explain processes and ideas of math concepts.

For example, in the "Range and Interquartile Range" lesson, students evaluate the descriptive statistics a sample student used to describe a data set. They explain whether the sample student is correct or incorrect. In the lesson "Using Ratio Notation," students construct an argument to explain whether a given ratio is a fraction. Students must use the definitions of ratios and fractions to construct a valid argument.





Students in Edgenuity courses are taught to evaluate their performance by asking themselves questions such as:

- Did I check my answer?
- How do I know the answer to the question is correct?
- If my answer isn't correct, what could I do differently?

Additionally, at critical points in instruction, students are asked to respond to open-ended prompts. Once students have constructed their responses, they are given a model answer and asked to compare their own writing to the model, using a checklist to identify the elements of the model that they included in their own answer. By comparing their response to a model and completing the checklist, they critically assess their own work and identify strengths and weaknesses in their thinking and communication.

How Edgenuity Mathematics Courses Teach Students to Model with Mathematics

As discussed above, Edgenuity tasks and assignments require students to create verbal, symbolic, and graphical models that solve complex problems that arise in everyday life, society, and the workplace.

Students regularly use visual models (e.g., number lines, area models, manipulatives) to illustrate real-world contexts for mathematical concepts. They also model mathematical and real-world problems with symbolic models (equations, expressions, inequalities). Students frequently connect multiple representations—such as tables and graphs—to model patterns and phenomena.

How Edgenuity Mathematics Courses Teach Students to Use Appropriate Tools Strategically

Edgenuity includes a wide array of virtual tools and manipulatives, as well as simulators, to help students explore concepts and verify solutions. Students select tools depending on the context of the task.

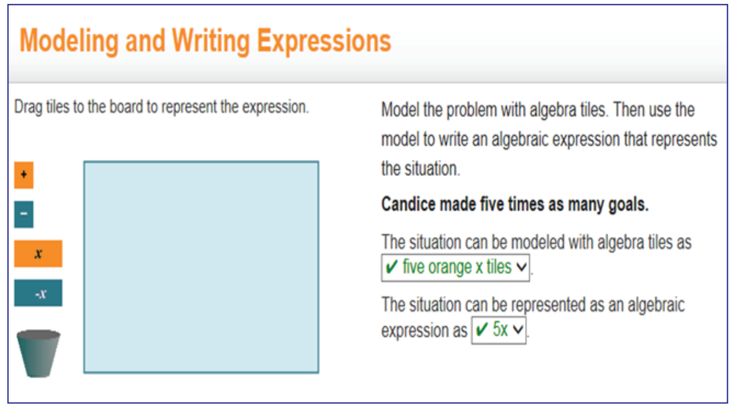
Calculators are available in Edgenuity courses as a matter of course in grade 8 and above (including a graphing calculator, matrix calculator, statistics calculator, and regression calculator at higher levels). Strategic use of calculators is present in grades 6 and 7. In all grades, students learn when technology can be used to make procedures more efficient and when mental math or paper and pencil work are more logical approaches.

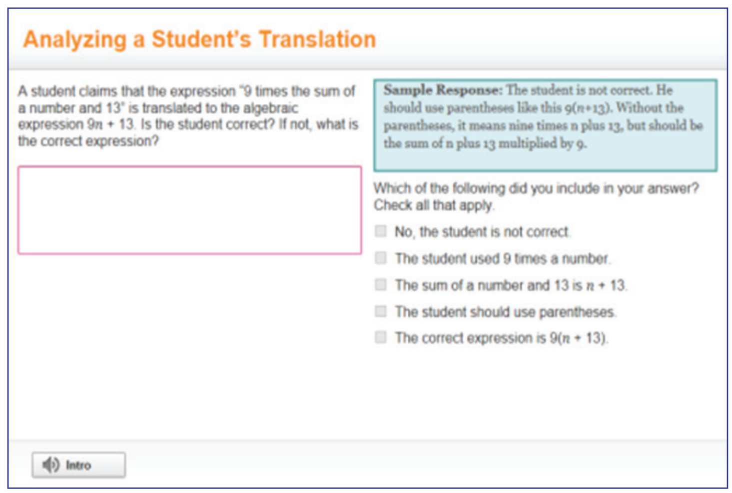
How Edgenuity Mathematics Courses Teach Students to Attend to Precision

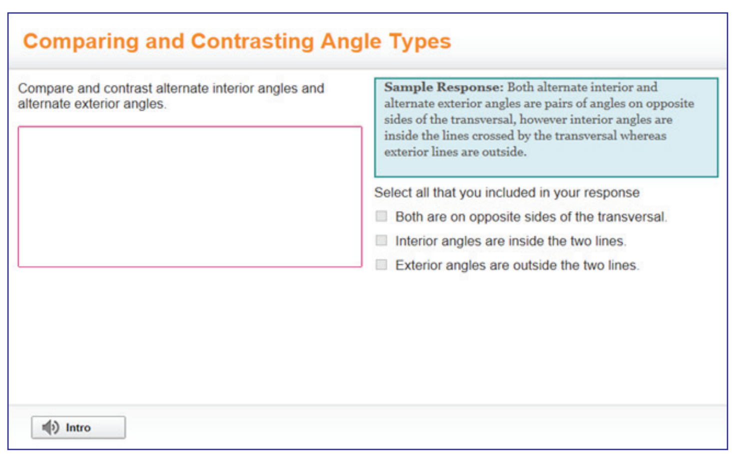
Edgenuity's on-screen teachers focus on clarity and accuracy of processes and outcomes in problem-solving. Not only are students expected to provide clear definitions, correctly identify units of measure, label axes accurately, use exact calculations, and use specific exemplars when explaining problems, but they are also required to investigate, formulate, and justify their thoughts.

For example, in a grade 7 lesson, students analyze expressions written verbally. They recall the terminology related to mathematical operations and use them to translate words into an expression. In this example, students determine whether the expression was converted correctly. If it was not, students must explain what it should be.

In the grade 8 course, students also answer with short responses using mathematical terms. For example, in the lesson "Transversals," students compare and contrast types of angles. They use angle terminology to describe the difference between angles inside and outside of a transversal.





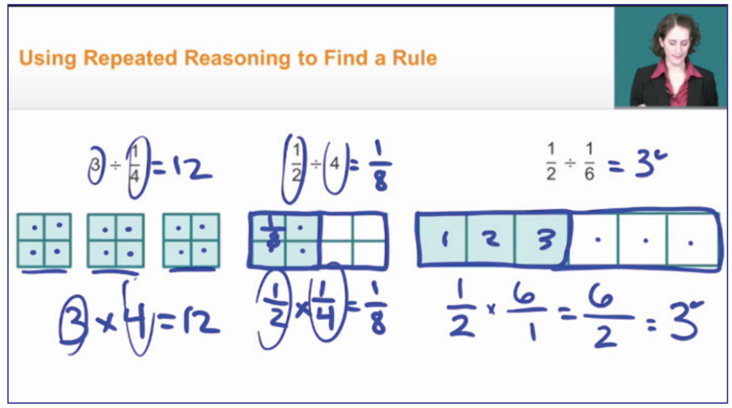


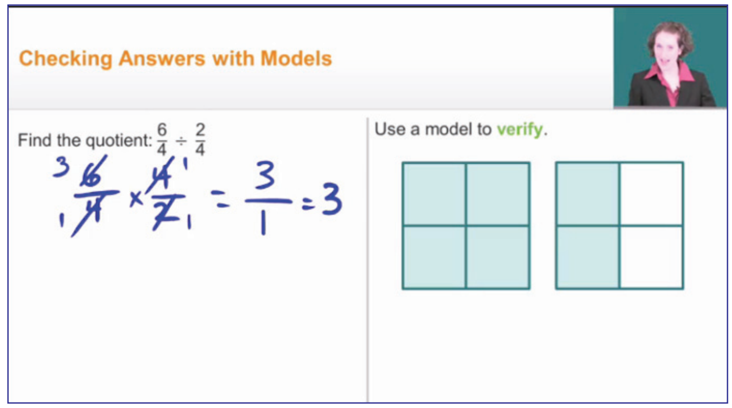
How Edgenuity Mathematics Courses Teach Students to Make Use of Structure

As stated above, Edgenuity courses help students look at problems and identify the numeric, geometric, and algebraic structures that are needed to solve them. To guide this process, on-screen teachers ask strategic questions to help students notice connections. For example:

- What does this make you think of?
- What other scenarios can you connect with this?
- When do you use these mathematics skills outside of school?
- Why is this expression equivalent to this other expression?
- Why does this procedure work?
- Why does this pattern exist?
- How is this problem similar to or different from others?

How Edgenuity Mathematics Courses Teach Students to Look for and Express Repeated Reasoning





Edgenuity mathematics courses require students to look for, construct, and make generalizations about patterns. On-screen instructors model how to notice patterns in shapes, calculations, and processes. They also point out shortcuts for solving problems.

For example, in a grade 6 lesson, an on-screen instructor models how to find quotients for several fraction division problems. The on-screen instructor states that when you ask, “How many ¼ are in 3?” you're basically counting four fourths, three times. Multiplying 3 by 4 is a more efficient way to count all the fourths. The on-screen instructor then asks students to find the quotients of ½ ÷ 4 and ½ ÷ ⅙. For all three problems, the on-screen instructor points out how the dividend stayed the same but the divisor changed (i.e., ¼ changed to 4; 4 changed to ¼; ⅙ changed to 6 or 6⁄1) when multiplying. She shows that she flipped the numerator and denominator. Based on this repeated reasoning, students make the generalization that they can divide fractions by multiplying the dividend by the reciprocal of the divisor. Finally, the instructor teaches students how to use an area model to verify the quotient.

Students also complete assignments and tasks where they draw conclusions about patterns and figure out the conditions in which they occur. Self-check questions ensure that students consistently check the reasonableness of their answers. For example, in a lesson on area, students are asked, “How many one-inch square tiles fit into a 3x5-inch rectangle? Into a 3x6-inch rectangle? A 3x7-inch rectangle? What conclusion could you propose about the relationship between a rectangle’s dimensions and its area?”

Conclusion

Edgenuity's online and blended courses translate the best mathematics research in online learning, neuroscience, pedagogy, educational psychology, and instructional design into its mathematics courses. Explicit instruction that addresses misconceptions, establishes clear learning objectives, balances conceptual and procedural knowledge, provides precise vocabulary instruction, offers clear instruction with examples and modeling, incorporates practice with scaffolded support and feedback, and incorporates reviews spread out over time ensure students know and master academic content.

Instruction is designed to build students' capacity to think critically and solve complex problems relating to numbers and operations, algebra, statistics and probability, and geometry. Delivery of rich core content and interactive assignments and tools enables students to apply knowledge in innovative ways. In addition, explicit metacognitive instruction helps students master core academic content and refine their mathematical practice—enabling them to make sense of problems, reason abstractly, construct viable arguments, model with mathematics, use appropriate tools strategically, attend to precision, make use of structure, and express repeated reasoning while learning. Edgenuity mathematics courses enhance students' ability to remember what they learned by breaking instruction into small chunks, pacing instruction properly, and capitalizing on multiple representations and graphic organizers to present information and connect known knowledge to new knowledge.

With over 500,000 enrollments each year, Edgenuity provides the tools and resources to help improve mathematical learning trajectories. For case studies and success stories describing how Edgenuity has met the diverse needs of students across a range of circumstances, please visit www.edgenuity.com/curriculum-research/research.

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